PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. VI. 1867–68. No. 75.

Monday, 16*th December* 1867.

PROFESSOR LYON PLAYFAIR, C.B., Vice-President, in the Chair.

The following Communications were read:—

1. On Polyzomal Curves, otherwise the Curves $\sqrt{U} + \sqrt{V} + \&c. = 0$.

By Professor Cayley. Communicated by Professor Tait.

If U, V, &c., are rational and integral functions $(*)(x, y, z)^r$, all of the same degree r, in regard to the co-ordinates (x, y, z), then $\sqrt{U} + \sqrt{V} +$ &c. is a polyzome, and the curve $\sqrt{U} + \sqrt{V} +$ &c. $= 0$ a polyzomal curve. Each of the curves $\sqrt{U} = 0$, $\sqrt{V} = 0$, &c. (or say the curves $U = 0$, $V = 0$, &c.), is on account of its relation of circumscription to the curve $\sqrt{U} + \sqrt{V} +$ &c. $= 0$, considered as a girdle thereto (ζωμα), and we have thence the term "zome" and the derived expressions "polyzome," "zomal," &c. If the number of the zomes $\sqrt{U}$, $\sqrt{V}$, &c. be $= \nu$, then we have a ν-zome, and corresponding thereto a ν-zomal curve; the curves $U = 0$, $V = 0$, &c., are the zomal curves or zomals thereof. The cases $\nu = 1$, $\nu = 2$, are not, for their own sake, worthy of consideration; it is in general assumed that ν is $= 3$ at least. It is sometimes convenient to write the general equation in the form $\sqrt{lU} +$ &c. $= 0$, where l, &c., are constants. The memoir contains researches in regard to the general ν-zomal curve; the branches thereof, the order of the curve, its singularities, class, &c.; also in regard to the ν-zomal curve $\sqrt{l(\Theta + L\Phi)} + \&c. = 0$, where the zomal curves $\Theta + L\Phi = 0$, all pass through the points of intersection of the same two curves $\Theta = 0$, $\Phi = 0$ of the orders r and $r-s$ respectively; included herein we have the theory of the depression of order as arising from the ideal factor or factors of a branch or branches. A general theorem is given of "the decomposition of a tetrazomal curve," viz., taking the equation to be $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} + \sqrt{pT} = 0$; then if U, V, W, T are in involution, that is, connected by an identical equation $\mathrm{a}\,U + \mathrm{b}\,V + \mathrm{c}\,W + \mathrm{d}\,T = 0$, and if l, m, n, p, satisfy the condition $\frac{l}{\mathrm{a}} + \frac{m}{\mathrm{b}} + \frac{n}{\mathrm{c}} + \frac{p}{\mathrm{d}} = 0$, the tetrazomal curve breaks up into two trizomal curves, each expressible by means of any three of the four functions U, V, W, T; for example in the form $\sqrt{l'U} + \sqrt{m'V} + \sqrt{p'T} = 0$. If, in this theorem, we take $p = 0$, then the original curve is the trizomal $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$, T is any function $= \frac{1}{\mathrm{d}}(\mathrm{a}\,U + \mathrm{b}\,V + \mathrm{c}\,W)$, where, considering l, m, n as given, a, b, c are quantities subject only to the condition $\frac{l}{\mathrm{a}} + \frac{m}{\mathrm{b}} + \frac{n}{\mathrm{c}} = 0$, and we have the theorem of "the variable zomal of a trizomal curve," viz., the equation of the trizomal $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$, may be expressed by means of any two of the three functions U, V, W, and of a function T determined as above, for example in the form $\sqrt{l'U} + \sqrt{m'V} + \sqrt{p'T} = 0$; whence also it may be expressed in terms of three new functions T, determined as above. This theorem, which occupies a prominent position in the whole theory, was suggested to me by Mr Casey's theorem, presently referred to, for the construction of a bicircular quartic as the envelope of a variable circle.

In the ν-zomal curve $\sqrt{l(\Theta + L\Phi)} + \&c. = 0$, if $\Theta = 0$ be a conic, $\Phi = 0$ a line, the zomals $\Theta + L\Phi = 0$, &c., are conics passing through the same two points $\Theta = 0$, $\Phi = 0$, and there is no real loss of generality in taking these to be the circular points at infinity—that is, in taking the conics to be circles. Doing this, and using a special notation $\mathsf{A}^0 = 0$ for the equation of a circle having its centre at a given point A, and similarly $\mathsf{A} = 0$ for the equation of an evanescent circle, or say of the point A, we have the ν-zomal curve $\sqrt{l\mathsf{A}^0} + \&c. = 0$, and the more special form $\sqrt{l\mathsf{A}} + \&c. = 0$. As regards the last mentioned curve, $\sqrt{l\mathsf{A}} + \&c. = 0$, the point A, to which the equation $\mathsf{A} = 0$ belongs, is a focus of the curve, viz., in the case $\nu = 3$, it is an ordinary focus, and in the case $\nu > 3$, it is a special kind of focus, which, if the term were required, might be called a foco-focus; the memoir contains an explanation of the general theory of the foci of plane curves. For $\nu = 3$, the equation $\sqrt{l\mathsf{A}} + \sqrt{m\mathsf{B}} + \sqrt{n\mathsf{C}} = 0$ is really equivalent to the apparently more general form $\sqrt{l\mathsf{A}^0} + \sqrt{m\mathsf{B}^0} + \sqrt{n\mathsf{C}^0} = 0$. In fact, this last is in general a bicircular quartic, and, in regard to it, the before-mentioned theorem of the variable zomal becomes Mr Casey's theorem, that "the bicircular quartic (and, as a particular case thereof, the circular cubic) is the envelope of a variable circle, having its centre on a given conic, and cutting at right angles a given circle." This theorem is a sufficient basis for the complete theory of the trizomal curve $\sqrt{l\mathsf{A}^0} + \sqrt{m\mathsf{B}^0} + \sqrt{n\mathsf{C}^0} = 0$; and it is thereby very easily seen that the curve $\sqrt{l\mathsf{A}^0} + \sqrt{m\mathsf{B}^0} + \sqrt{n\mathsf{C}^0} = 0$ can be represented by an equation $\sqrt{l'\mathsf{A}'} + \sqrt{m'\mathsf{B}'} + \sqrt{n'\mathsf{C}'} = 0$. But for $\nu > 3$, this is not so, and the curve $\sqrt{l\mathsf{A}} + \&c. = 0$ is only a particular form of the curve $\sqrt{l\mathsf{A}^0} + \&c. = 0$; and the discussion of this general form is scarcely more difficult than that of the special form $\sqrt{l\mathsf{A}} + \&c. = 0$, included therein. The investigations in relation to the theory of foci, and in particular to that of the foci of the circular cubic and bicircular quartic, precede in the memoir the theories of the trizomal curve $\sqrt{l\mathsf{A}^0} + \sqrt{m\mathsf{B}^0} + \sqrt{n\mathsf{C}^0} = 0$, and of the tetrazomal curve $\sqrt{\ \mathsf{A}^0} + \sqrt{m\mathsf{B}^0} + \sqrt{n\mathsf{C}^0} + \sqrt{p\mathsf{D}^0} = 0$, to which the concluding portions relate. I have, accordingly, divided the memoir into four parts, viz., these are Part I., On Polyzomal Curves in general; Part II., Subsidiary Investigations; Part III., On the Theory of Foci; and Part IV., On the Trizomal and Tetrazomal Curves, where the zomals are circles. There is, however, some necessary intermixture of the theories treated of, and the arrangement will appear more in detail from the headings of the several articles. The paragraphs are numbered continuously through the memoir. There are four Annexes, relating to questions which it seemed to me more convenient to treat of thus separately.

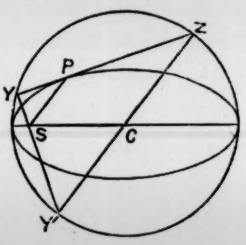
It is right that I should explain the very great extent to which, in the composition of the present memoir, I am indebted to Mr Casey's researches. His paper "On the Equations and Properties (1.) of the System of Circles touching three circles in a plane; (2.) of the System of Spheres touching four spheres in space; (3.) of the System of Circles touching three circles on a sphere; (4.) on the System of Conics inscribed in a conic, and touching three inscribed conics in a plane," was read to the Royal Irish Academy, April 9, 1866, and is published in their "Proceedings." The fundamental theorem for the equation of the pairs of circles touching three given circles was, previous to the publication of the paper, mentioned to me by Dr Salmon, and I communicated it to Professor Cremona, suggesting to him the problem solved in his letter of March 3, 1866, as mentioned in my paper, "Investigations in connection with Casey's Equation," "Quart. Math. Journal," t. viii. 1867, pp. 334–341, and as also appears, Annex No. IV. of the present memoir.

In connection with this theorem, I communicated to Mr Casey, in March or April 1867, the theorem No. 164 of the present memoir, that for any three given circles, centres A, B, C, the equation $BC\sqrt{\mathsf{A}^0} + CA\sqrt{\mathsf{B}^0} + AB\sqrt{\mathsf{C}^0}$ belongs to a Cartesian. Mr Casey, in a letter to me dated 30th April 1867, informed me of his own mode of viewing the question as follows:—"The general equation of the second order $(a, b, c, f, g, h)(\alpha, \beta, \gamma)^2$, where α, β, γ are circles, is a bicircular quartic. If we take the equation $(a, b, c, f, g, h)(\lambda, \mu, \nu)^2$ in tangential co-ordinates (that is, when λ, μ, ν are perpendiculars let fall from the centres of α, β, γ on any line), it denotes a conic; denoting this conic by F, and the circle which cuts α, β, γ orthogonally by J, I proved that, if a variable circle moves with its centre on F, and if it cuts J orthogonally, its envelope will be the bicircular quartic whose equation is that written down above;" and among other consequences, he mentions that the foci of F are the double foci of the quartic, and the points in which J cuts F single foci of the quartic, and also the theorem which I had sent him as to the Cartesian, and he refers to his paper on bicircular quartics as then nearly finished. An abstract of the paper as read before the Royal Irish Academy, 10th February 1867, and published in their "Proceedings," pp. 44, 45, contains the theorems mentioned in the letter of 30th April, and some other theorems. It is not necessary that I should particularly explain in what manner the present memoir has been, in the course of writing it, added to or altered, in consequence of the information which I have thus had of Mr Casey's researches; it is enough to say that I have freely availed myself of such information, and that there is no question as to Mr Casey's priority in anything which there may be in common in his memoir on Bicircular Quartics and in the present memoir.

2. Note on the Hodograph. By Professor Tait.

The object of the present Note is to show, by a few examples (of which, however, the last is the only one of any real importance), how easily the geometrical ideas supplied by Hamilton's beautiful invention of the Hodograph enable us to dispense with analytical processes in the establishment of some of the fundamental propositions connected with the motion of a single particle, besides many others which are merely curious; and also how they help us to understand the full bearing of some of the analytical methods. Some of the simplest of such geometrical investigations are given in "Tait and Steele's Dynamics of a Particle," and will not be reproduced here; though a few of the results will be assumed,—as, for instance, that when the acceleration is directed to a fixed point, and varies inversely as the square of the distance from it, the hodograph is a circle, and the path a conic section, of which the point is a focus.

1. If the figure represent an ellipse and its auxiliary circle, it is known that the circle may be considered as the hodograph corresponding to planetary motion in the ellipse, but turned through a right angle. In fact, if YPZ be a tangent to the ellipse at P, SY′ is proportional to the velocity at P, and perpendicular to it in direction. The actual velocity bears to SY′ the ratio of μ to ha, in the usual notation.



Hence the tangent at Y′ is perpendicular to SP (the direction of acceleration), and thus we have an immediate proof that SP is parallel to Y′CZ. But by this means we also get at once, and without analysis, the two well-known and peculiar first integrals, in the form

$$\dot{x} = -\frac{\mu}{h}\frac{y}{r}, \quad \dot{y} = \frac{\mu}{h}\left(\frac{x}{r} + e\right),$$

which cannot be directly deduced from the equations of acceleration

$$\ddot{x} = -\frac{\mu x}{r^3}, \quad \ddot{y} = -\frac{\mu y}{r^3}.$$

[The equation of the orbit is, of course,

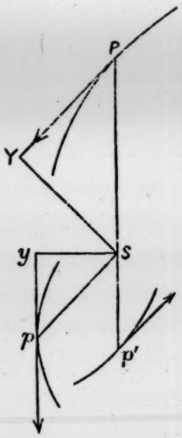
$$h = x\dot{y} - y\dot{x} = \frac{\mu}{h}\left(r + ex\right),$$

from which we see that

$$h^2 = \mu a\left(1 - e^2\right)\Big].$$

2. The only central orbits whose hodographs also are described as central orbits, are those in which the acceleration varies directly as the distance from the centre.

Let S be the centre, P any point in the path, p the corresponding point in the hodograph, p' that in the hodograph of the hodograph. Then Sp' is parallel to the tangent at p, which again is parallel to SP. Hence PSp' is a straight line. Also, since p belongs (by hypothesis) to a central orbit, the tangent at p' is parallel to Sp, *i. e.*, to the tangent at P. Hence the locus of p' is similar to that of P, and therefore Sp' is *proportional to* SP. But Sp' represents the acceleration at P. Hence the proposition.



3. If Π be the acceleration in a central orbit, Π′ that required for the description of the hodograph as a central orbit; h, h', the moments of momentum, and r, r', the radii vectores in the two orbits,

$$\Pi\Pi' = \frac{h'^2}{h^2}rr'.$$

In the figure above let SY $= \varpi$ and S$y = \varpi'$ be the perpendiculars from S on the tangents at P and p, ρ and ρ' the radii of curvature at P and p, then

$$\frac{r}{\varpi} = \frac{r'}{\varpi'} .$$

Also the velocity at p is

$$\Pi = \frac{h\rho'}{r^3} .$$

But, since we have $$\Pi = r' . \frac{r'}{\rho} . \frac{r}{\varpi}$$

(as we see by expressing it in terms of the angular velocity of Sp), if Sp' be called r'', we have

$$\Pi' = r'' . \frac{r''}{\rho'} . \frac{r'}{\varpi'} .$$

Hence, as $$\varpi r' = h , \varpi' r'' = h' ,$$

$$\Pi\Pi' = \frac{h}{r^2} . \frac{r' r''^2}{\varpi'} = \frac{h}{r^2} \frac{r' h'^2}{\varpi'^3} = \frac{h'^2}{h^2} r r' .$$

Or, more simply, if v be the velocity in the orbit, we have, by expressing the centrifugal force in terms of the normal component of the acceleration,

$$\frac{v^2}{\rho} = \Pi \frac{\varpi}{r} .$$

Hence $$\frac{h^2}{\rho} = \Pi \frac{\varpi^3}{r} .$$

[This is the well-known formula

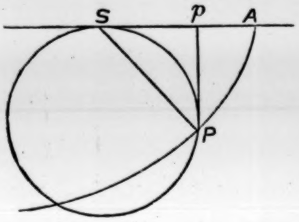
$$\Pi = \frac{h^2}{\varpi^3} \frac{d\varpi}{dr} .]$$

Thus $$\Pi\Pi' = \frac{h^2 r}{\varpi^3 \rho} . \frac{h'^2 r}{\varpi'^3 \rho'} = \frac{h'^2}{h^2} r r' ,$$

because from $$r\varpi' = r'\varpi = h$$

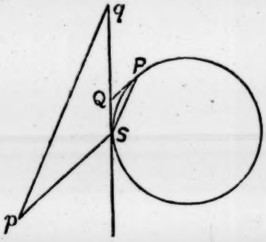
we have at once $$r^2 r'^2 = \varpi\varpi' \rho\rho' .$$

4. Again, if the hodograph be a circle described with uniform angular velocity about a point in its circumference, the path is the cycloidal brachistochrone.



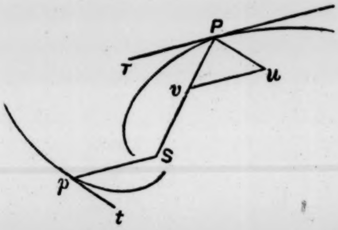
For, if AP be the cycloid described by the point P of the circle SP rolling uniformly on the line AS, the velocity at P is proportional to SP, and the direction of motion is perpendicular to SP. Hence the hodograph (turned through a right angle in its own plane) may be represented by the circle SP, described with uniform angular velocity about the point S. That the motion is due to constant acceleration perpendicular to AS is obvious from the fact that, if P*p* be drawn perpendicular to AS, $SP^2 \propto Pp$.

5. If the orbit be central, and be a circle described about a point in its circumference, the hodograph is a parabola described about the focus with angular velocity proportional to the radius vector.



For, if S be the centre of force, P the point in its circular orbit, *p* the corresponding point of the hodograph: *qp*, the tangent to the hodograph at *p*, must be parallel to SP; and, therefore, if SQ*q* be the tangent at S, the triangle *p*S*q* (being similar to PSQ) is isosceles. Thus the locus of *p* is a parabola. Also the angular velocity of S*p*, being the same as that of PQ, is double that of SP, and is, therefore, inversely as SP². But the length of S*p* is inversely as the perpendicular from S upon PQ, *i.e.*, inversely as SP².

6. A point describes a logarithmic spiral with uniform angular velocity about the pole—find the acceleration.



Since the angular velocity of SP and the inclination of this line to the tangent are each constant, the linear velocity of P is as SP. Take a length PT, equal to *e* SP, to represent it. Then the hodograph, the locus of *p*, where S*p* is parallel, and equal, to PT, is evidently another logarithmic spiral similar to the former, and described with the same uniform angular velocity. Hence *pt*, the acceleration required, is equal to *e* S*p*, and makes with S*p* an angle equal to SPT. Hence, if P*u* be drawn parallel and equal to *pt*, and *uv* parallel to PT, the whole acceleration P*u* may be resolved into P*v* and *vu;* and P*vu* is an isosceles triangle, whose base angles are each equal to the angle of the spiral. Hence P*v* and *vu* bear constant ratios to P*u*, or to SP or PT.

The acceleration, therefore, is composed of a central attractive part proportional to the distance, and a tangential retarding part proportional to the velocity.

And, if the resolved part of P's motion parallel to any line in the plane of the spiral be considered, it is obvious that in it also the acceleration will consist of two parts—one directed towards a point in the line (the projection of the pole of the spiral), and proportional to the distance from it, the other proportional to the velocity, but retarding the motion.

Hence a particle which, unresisted, would have a simple harmonic motion, has, when subject to resistance proportional to its velocity, a motion represented by the resolved part of the spiral motion just described.

If α be the angle of the spiral, ω the angular velocity of SP, we have evidently

$$\mathrm{PT} \, . \sin \alpha = \mathrm{SP} \, . \, \omega .$$

Hence

$$\mathrm{P}v = \mathrm{P}u = pt = \frac{\mathrm{PT}^2}{\mathrm{SP}} = \frac{\omega}{\sin \alpha} \mathrm{PT} = \frac{\omega^2}{\sin^2 \alpha} \mathrm{SP} = n^2 . \mathrm{SP} \text{ (suppose)}$$

$$\text{and } vu = 2\mathrm{P}v \, . \cos \alpha = \frac{2\omega \cos \alpha}{\sin \alpha} \mathrm{PT} = k \, . \, \mathrm{PT} \text{ (suppose.)}$$

Thus the central force at unit distance is $n^2 = \frac{\omega^2}{\sin^2 \alpha}$, and the coefficient of resistance is $k = \frac{2\omega \cos \alpha}{\sin \alpha}$.

The time of oscillation is evidently $\frac{2\pi}{\omega}$; but, if there had been no resistance, the properties of simple harmonic motion show that it would have been $\frac{2\pi}{n}$; so that it is increased by the resistance in the ratio cosec α : 1, or $n : \sqrt{n^2 - \frac{k^2}{4}}$.

The rate of diminution of SP is evidently

$$\text{PT.} \cos a = \frac{\omega \cos a}{\sin a} \text{ SP} = \frac{k}{2} \text{SP};$$

that is, SP diminishes in geometrical progression as time increases, the rate being $\frac{k}{2}$ per unit of time per unit of length. By an ordinary result of arithmetic (compound interest payable every instant) the diminution of log. SP in unit of time is $\frac{k}{2}$.

This process of solution is only applicable to resisted harmonic vibrations when n is greater than $\frac{k}{2}$. When n is not greater than $\frac{k}{2}$ the auxiliary curve can no longer be a logarithmic spiral, for the moving particle never describes more than a finite angle about the pole. A curve, derived from an equilateral hyperbola, by a process somewhat resembling that by which the logarithmic spiral is deduced from a circle, must be introduced; and then the geometrical method ceases to be simpler than the analytical one, so that it is useless to pursue the investigation farther, at least from this point of view.

3. On the Antiquity of Intellectual Man: from a Practical and Astronomical point of view. By Professor C. Piazzi Smyth.

The author of this paper confined himself entirely to the use of *contemporary* data; and amongst these, the only kinds found available for the very earliest ages, were buildings of one kind or another.

Examining these again, after their first approximate arrangement in chronological relative order by well-known archæologists, it was discovered that there resided a peculiar astronomical opposition between the chief structures in the valley of the Euphrates and those in the valley of the Nile.

The final contest for antiquity amongst all the archæological known remains of the whole earth seemed to lie entirely with the

Euphrates and Nile series of buildings; the palm being assigned at last to the Great Pyramid beginning of the latter.

This particular monument, being then still further examined for the manner of its origination and appearance, was found to be in direct violation of what has often been laid down elsewhere to be the rule or natural law of all purely human work. And as such interference with a natural law, is in so far one of the ordinary definitions of a miracle, a yet more stringent examination of the monument was made; and resulted in the discovery, partly by some of the author's friends, of a continued and consistent series of geometrical and physical data, infinitely above the knowledge of all mankind in the days when the Great Pyramid was constructed; and therefore in so far strengthening the conclusion for a miracle, already otherwise obtained.

First, therefore, beyond compare, of all known human architectural remains, in point of both time, moral purity, mechanical excellence, and philosophical knowledge; and dating itself astronomically from no further back than 2170 B.C., the author thought that the continued study of this Great Pyramid building was likely to shed a most unexpected amount of light on the doctrine of miracles in general; as well as on what was the manner of appearance of the first traces of a high order of human civilisation in the present post-diluvian history of the world.

Monday, 6th January 1868.

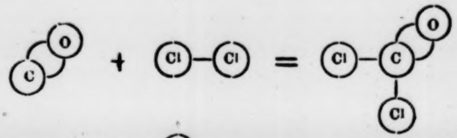
Dr CHRISTISON, Vice-President, in the Chair.

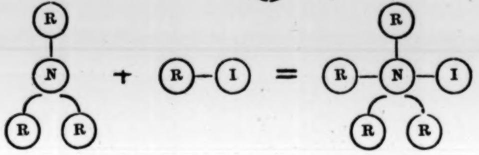
The Chairman, in presenting the Keith Medal to Professor Piazzi Smyth, read the following Minute of Council relative to the award of the prize:—" The Committee, having examined the papers presented to the Society within the last biennial period, resolve to recommend the Council to award the Keith prize to the communication from Professor Piazzi Smyth, entitled 'Notes of Recent Measurements at the Great Pyramid;' and the Committee desire to record their opinion that the energy, self-sacrifice, and skill displayed by Professor Smyth in that vast series of measurements, the details of which are appended to the second volume of his 'Life and Work at the Great Pyramid,' fully entitle him to the highest testimony which the Council has in its power to confer." The Chairman then handed the Keith Medal to Professor Smyth, remarking that the Council offered no opinion on the important theories he had enunciated, but recognised the valuable character of the measurements he had made.

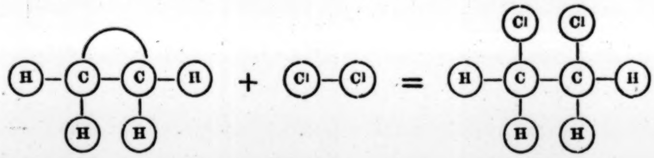
The following Communications were read:—

1. On the Changes produced by direct Chemical Addition on the Physiological Action of certain Poisons. By Drs A. Crum Brown and Thomas R. Fraser.

In this paper the authors communicate the first results of an attempt to discover the relation which must exist between the chemical constitution and the physiological action of a substance. As the chemical constitution of the majority of physiologically active substances is unknown, they investigate the subject by examining the physiological action of a substance before and after the performance upon it of a definite chemical operation, introducing a known change into its constitution. The question is thus reduced to a problem in what may be called a chemico-physical calculus of finite variations. Thus, if the chemical constitution be represented by C, the physiological action, P, is some unknown function of C, say fC. In order to find f a change is produced on C, by which it becomes $C + \Delta C$, and the corresponding change of physiological action from fC to $fC + \Delta f \cdot C$ is investigated. We here know ΔC, fC, and $\Delta f \cdot C$; and by finding their relations for a sufficiently large number of values of C (even where these are unknown), and, by varying ΔC, the function f may be determinded. The change of constitution represented by ΔC must be a simple and unambiguous one. There are two kinds of operation to choose between —replacement and addition; and the authors select as the first subject of inquiry the effect of addition—that is, such chemical change as increases the active atomicity of atoms or radicals. The following may be given as simple examples of such operations:—







Their reason for this selection is, that, as far as it is possible to judge from previously known facts, replacement does not produce nearly so great a change of physiological action as addition does. Thus, comparing the action of carbonic oxide and carbonic acid, hydrocyanic acid and methylamine, arsenious and kakodylic acids, strychnia and brucia, and the salts of the ammonium bases derived from them, it may be seen that addition, in many cases at least, diminishes or removes physiological activity. This comparison leads to a suspicion that physiogical activity is in some way connected with *chemical condensation*, by which term the authors mean susceptibility of addition, whether the addition takes place by the increase of the atomicity of an atom, or of a group of atoms. This suspicion receives some confirmation from the fact, that such of the stable combinations of pentatomic arsenic and antimony, as have been examined physiologically, are stated to be inert, while all the soluble compounds of triatomic arsenic and antimony are active; similarly, the aromatic bodies are, as a rule, more active than the corresponding fatty bodies. The occurrence, however, of such poisons as alcohol, oxalic acid, and corrosive sublimate among saturated substances, and of comparatively inert condensed compounds, such as benzoic acid and salicine, shows that condensation is not the only condition of physiological activity.

The statements of Stahlschmidt and Schroff, in reference to the action of the salts of methyl strychnia, induced the authors to turn their attention, in the first place, to the effect of the addition of iodide of methyl to the natural alkaloids. As the iodides of the complex ammoniums thus produced are, in most cases, sparingly soluble in water, they have also examined the action of the corresponding sulphates.

The poisonous alkaloids thus examined, and included in this paper, are strychnia, brucia, thebaia, codeia, morphia, and nicotia. The authors give details of the processes followed in obtaining the iodide and the sulphate of the methyl-derivatives of these bases, and they describe their physiological effects.

Twelve grains of iodide of methyl-strychnium,* subcutaneously administered, produced no effect on a rabbit, weighing three pounds. Fifteen grains were recovered from, after symptoms; and twenty grains was a fatal dose. When exhibited by the stomach, twenty grains of this compound did not cause any symptom; while the same rabbit was rapidly killed by one-tenth of a grain of strychnia, given in exactly the same way. Twenty grains of iodide of methyl-strychnium contain about fourteen grains of strychnia.

The sulphate of methyl-strychnium, being a much more soluble salt than the iodide, was found to have a much smaller poisonous dose. One grain was fatal to a rabbit, by subcutaneous exhibition. Eight-tenths of a grain were recovered from, while five-tenths did not cause any symptom. The rabbit that recovered after the administration of eight-tenths of a grain of sulphate of methyl-strychnium, died shortly after one-twentieth of a grain of strychnia was injected under the skin.

Both the iodide and the sulphate of methyl-strychnium produced symptoms altogether different from those of strychnia. There were no convulsions, nor was there the slightest exaggeration of the reflex function; the symptoms were those of paralysis, and death was produced by the asphyxia that this occasioned. The authors further investigated this action by localised poisoning in frogs; and they have demonstrated that iodide and sulphate of methyl-strychnium paralyse the peripheral terminations (end-organs) of the motor nerves, and, therefore, possess exactly the same action as curare (wourali) has.

Brucia and thebaia act in the same way as strychnia, and it was found that iodide and sulphate of methyl-brucium and methyl-thebaium have the same action as the analogous strychnia compounds. The fatal dose of iodide of methyl-brucium was found to be very much the same as that of the corresponding compound of strychnia; a larger dose was, however, necessary to produce death with sulphate of methyl-brucium, than with sulphate of methyl-strychnium. Iodide of methyl-thebaium, being more soluble in warm water, has a smaller fatal dose than the iodides of methyl-strychnium and methyl-brucium. Six grains produced no effect when injected under the skin of a rabbit; eight grains caused symptoms, which were recovered from; and death occurred eleven minutes after the injection of ten grains. Eight grains of iodide of methyl-thebaium contained about five grains and a half of thebaia, and, for a rabbit, the fatal dose of this alkaloid is one-fifth of a grain.

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* The action of iodide of ethyl-strychnium was also examined, and found to be the same as that of iodide of methyl-strychnium.

Among the opium alkaloids, codeia ranks next to thebaia in activity. It was found by the authors, that six grains of iodide of methyl-codeium dissolved in warm warer, and injected under the skin of a rabbit, caused no effect. Ten grains, however, was an almost fatal dose, and this contains about twelve times as much codeia as would kill a rabbit. It was also found that the fatal dose of sulphate of methyl-codeium is not very different from that of the iodide. Neither of these compounds possess the usual convulsant action of codeia; and as this alkaloid has but a feeble soporific action, it was difficult to determine how far this was modified by the direct chemical addition of methyl compounds. The author also found that iodide and sulphate of methyl-codeium paralyse the motor nerve end-organs, an effect that is not produced by codeia itself.

Iodide of methyl-morphium is a very insoluble substance. The largest dose that could, therefore, be administered subcutaneously to a rabbit was twenty grains, and this large quantity produced no effect. Eight grains of morphia were, some days afterwards, exhibited in the same way to this rabbit; the result was a decided soporific effect, followed by epileptiform convulsions and death.

No effect was produced when iodide of methyl-morphium was administered to rabbits by the stomach, even in so large a dose as thirty grains.

Recognising the possible fallacies connected with experiments with such a substance on rabbits, the authors determined to observe the effect on man. One of themselves, accordingly, took, on one occasion, half a grain of iodide of methyl-morphium as a powder, and on another, one grain (containing about three-fourths of a grain of morphia), but on neither occasion was there observed the slightest soporific or other effect.

Four grains of sulphate of methyl-morphium produced decided narcotism on a rabbit, but no convulsive effect. Indeed, with this dose, and with various others that were given, paralysis appeared, and the authors have demonstrated that this symptom is due to an effect on the motor nerve end-organs.

Iodide of methyl-nicotium was obtained in the form of crystals extremely soluble in cold water. When given to rabbits by subcutaneous injection, a dose of five grains was perfectly inert; one of fifteen grains produced serious symptoms, followed by recovery: and a dose of twenty grains was fatal. The symptoms were principally distinguished from those of nicotia by the absence of convulsions; but no paralytic action on motor nerve end-organs was caused.

The authors have also investigated the action of iodide of methyl, and they obtained no evidence in support of the extremely improbable hypothesis, that some of the changes produced in the action of the substances they had examined might have been due to the addition of the physiological action of the methyl compounds. They conclude by discussing the possible causes of these modifications, by pointing out some of the practical applications of their results, and by promising to examine how far iodide of methyl may prove an antidote to the poisonous effects of these vegetable alkaloids, whose fatal dose it increases.

2. On the Burning Mirrors of Archimedes, and on the Concentration of Light produced by Reflectors. By John Scott, Esq. Communicated by Professor Kelland.

That the Roman ships were destroyed by burning glasses invented by Archimedes, is mentioned as a fact by most of the ancient writers, especially those who treat on mechanics, and their statements have been repeated by succeeding authors, without any doubts having been suggested until comparatively recent times. The earliest authorities on the subject are Diodorus Siculus, Lucian, Galen, Dion Cassius, and Pappus. It is much to be regretted that a work by the last named author on the Siege of Syracuse is now lost; but Zonares and Tzetzes, writers of the 12th century, in whose time it was extant, give quotations from it. That of the latter, translated pretty literally, runs thus:—"When Marcellus had placed the ships a bow shot off, the old man (Archimedes) constructed a sort of hexagonal mirror. He placed at proper distances from the mirror other smaller mirrors of the same kind, which were moved by means of their hinges and certain plates of metal. He placed it amid the rays of the sun at noon, both in summer and winter. The rays being reflected by this, a frightful fiery kindling was excited on the ships, and it reduced them to ashes, from the distance of a bow shot. Thus the old man baffled Marcellus, by means of his inventions."

At a later period, mirrors similar to that of Archimedes, appear to have engaged the attention of Baron Napier of Merchiston, and other mathematicians; but strange enough, the famous naturalist, Buffon, was the first to establish the practicability, and therefore the probability, of the achievement. He employed a combination of plane reflectors, consisting of ordinary looking-glasses, eight inches by six, attached to a single frame. With forty of these glasses he set on fire tarred beech at a distance of 66 feet. A plank smeared with tar and brimstone, was ignited at 126 feet, by 98 glasses. A combination of 128 glasses, with a clear sun, inflamed very suddenly a plank of tarred fir at 150 feet. In addition to these experiments made at Paris, about the beginning of April, others were made in summer, by which wood was kindled at 200 and 210 feet, and silver and other metals were melted at distances varying from 25 to 40 feet.

So much for the positive side of the question, let us now briefly consider the negative side.

Descartes and others treated the whole affair of the burning mirrors as a fiction, on the gratuitous assumption that the specula employed were single parabolic reflectors. A different and more valid reason for doubt has arisen from the circumstance, that

Polybius, Livy, and Plutarch, are silent respecting the burning mirrors, although they describe some of the other contrivances employed by Archimedes against the assailants. In reply to this objection, it has been stated, as an admitted axiom in estimating historical evidence, that the silence of one author is not sufficient to disprove an event of which another, even subsequent in point of time, has given a plain and consistent narrative. Besides, similar omissions by ancient historians are not unfrequent; and in this instance, it is the less surprising, since the invention had perished with the fall of Syracuse, and the death of the illustrious inventor, who, in the peculiar circumstances, must have endeavoured to keep from his contemporaries the secret of its construction.

Returning to Buffon's experiments, Peyrard appends to his translation of the works of Archimedes a memoir of his own, pointing out the defects of Buffon's combination of reflectors, viz.:—That the number of assistants required to adjust so many mirrors were apt to confuse one another—the operation itself was a tedious one, taking about half an hour for one hundred and sixty-eight mirrors—and, after all, the superposition of the light was imperfect. Again, after a short interval, a readjustment would be rendered necessary by the angular deviation of the rays, arising from the varying position of the sun. Peyrard suggests " that each mirror should be furnished with a telescope, and a somewhat complex apparatus, to render the adjustment more speedy and accurate, and by this means to diminish the number of assistants required. This remedy is admitted to be only partial. It is proposed, in what follows, to obviate the forementioned defects, by producing a solar reflecting machine, capable of being directed by one eye, guided by a single hand, and coinciding in its construction with the description in the fragment preserved by Tzetzes from Pappus.

It is stated that Archimedes contrived a hexagonal mirror, and placed at proper distances from it other smaller ones of the same kind. This peculiarity has no parallel in that of Buffon. Articles 11, 12, and 13, in the memoir, show how burning mirrors can be constructed, corresponding to the above description, free from the defects of Buffon, and capable, moreover, of darting the consuming rays instantaneously in any given direction, thereby affording a strong presumption that the accounts respecting the Archimedian Mirror are in the main authentic.

3. On the Great Pyramid of Gizeh, and Professor C. P. Smyth's Views concerning it. By A. D. Wackerbarth, A.M., of Upsala. Communicated by Professor Kelland.

The author gives a detailed statement of the theories of Professor Smyth, as given in the Transactions of this Society, Vol. XXIII. Part III. He then, after heartily commending the zeal and diligence of the Professor, brings forward objections to some of his views. 1. As to the metron or unit of linear measure. Mr Wackerbarth objects that this measure was utterly unknown to the ancient Egyptians—appearing in no Egyptian document or monument whatever, nor in any ancient writer who describes the condition of the Egyptians. Mr Wackerbarth collects from the materials most accessible the following table of weights and measures in use amongst the ancient Egyptians (corrected for the recent measure of the cubit of Karnak, made by Sir Henry James).

Long Measure.									English Inches.
1. ⲧⲏⲃ (Digitus),									0·73925
2	1. Κόνδυλος,								1·4785
4	2	1. ϣⲟⲡ (Palmus),							2·957
5	2½	1¼	1. ϭⲛⲁⲧϩ (Vola),						3·69625
6	3	1½	1⅕	1. Kubdeh,					4·4355
8	4	2	1⅗	1⅓	1. Διχάς,				5·914
11	5½	2¾	2⅕	1⅚	1⅜	1. Fitr,			8·13175
13	6½	3¼	2⅗	2⅙	1⅝	1 2/11	1. ⲉⲣⲧⲱ (Σπιθαμή),		9·61925
28	14	7	5⅜	4⅔	3½	2 6/11	2 2/13	1. ⲙⲁϩⲓ (cubit)	20·699

Itinerary Measure.			
1. ϩⲟⲟⲩⲧⲛ (Στάδιον),			606·875 ft. Eng., or 623 ft. Swedish.
60	1. ⲛⲟϩ ⲛ̀ⲣⲱϣ (Σχοῖνος),		6·9 English miles, or 1 Norwegian mile = 1/10 degree of equator.
150	2½	1. ⲁⲟⲩⲣ	17·2 English miles, or 2½ Norweg. miles = ¼ degree.

LAND MEASURE.

1. Ἄϱουϱα = (100 cubits)^2 = 10,000 square cubits = 0·683 English acres = 0·557 Sw. Tunnaland.

WINE MEASURE.

1. Ἀϱυστήϱ = about ½ pint English, or about $\frac{1}{10}$ Swedish kanna.

WEIGHT.

1. ⲉⲙⲛⲁ (Mina) = 8304 grains = 1·1863 lb. av., or 1·278 Swedish Skålpund.

2. The author then, finding no such measure as the metron, goes on to say:—

"It is really very unpleasant to reject a beautiful and ingeniously carried out theory which one wishes to believe, but I cannot, however, arrive at any other conclusion than that Professor Smyth's very interesting account of an ancient Egyptian standard of length, exactly equal to a ten-millionth of the earth's axis of rotation, and the entire system of weights and measures that he has deduced from this imaginary unit, requiring accurate knowledge of the earth's dimensions, figure, and density, several centuries before the age of Abraham, and the supposed connection of the English measures with these, are things purely mythical. On the other hand, that the pyramid and its contents were really the standards of the ancient Egyptian kingdom, is, or seems to me an opinion not destitute of probability; but I cannot think that those standards were constructed with any reference to the dimensions of the earth, but that they were arbitrarily chosen quantities, intended to represent on a tolerably large scale the size of those portions of the human body which their names indicate: *e.g.*, ⲙⲁϩⲓ (*cubitus*), the arm from the elbow to the point of the middle finger; ϣⲟⲡ *palmus*), the breadth of the hand, &c., &c.

"In fine, Professor Smyth's method, namely, that of multiplying or dividing by quantities, for the introduction of which it would be hard to give any satisfactory reason,—as, for example, the number 366, with which the base was divided, and which is neither the exact number of days in a sidereal year, nor anything else, and again, the very uncertain number, 5,672 (the earth's density), —appears to me in the highest degree dangerous. How deceitful results thus obtained can be, I shall now, in conclusion of this discussion, attempt to illustrate by a few examples; and before doing this, I beg to state that, entertaining the highest respect for Professor Smyth, I have not the least intention of making light of his work, though this method of reasoning, being analogous to the *reductio ad absurdum*, will unavoidably at times have something of that appearance.

"*Ex.* 1. It is a historical fact, that the present English weights and measures are not of any very great antiquity. At the time of the Norman conquest the yard was about 39·6 of the present English inches, a little longer than the French mètre; and the foot, accordingly, 13·2 modern inches, a little greater than the Paris foot. In the year 1101, King Henry I. determined the yard by the length of his own arm, and that is the determination which the present yard is intended to represent; and that yard, moreover, has never been, even in modern times, defined by any fraction of any of the earth's dimensions, but by its proportion to the length of the seconds' pendulum at 51½° latitude, and of this yard the English foot is the third part. Now, a degree of the equator is just 365,260·524 ft. Divide a thousandth part of that number by the length of the sidereal year in solar days, 365·256358, and we have 1·0000114; that is to say, if we take a thousandth of a degree of the equator, and divide it by the number of days in a sidereal year, we have an English foot as nearly as a powerful microscope can determine it. *And yet it is certain that this is purely accidental.*

"*Ex.* 2. If I take 10,000 times e, the base of the hyperbolic logarithms, and multiply it into the quantity, which in the lunar theory is called g,—that is, the ratio of the difference between the moon's and its ascending node's mean motions to the moon's mean motion,—and divide the earth's polar radius by the product, the result is the length of the pyramid's side. But are we to suppose that the Egyptians forty centuries ago were acquainted with the lunar theory, the earth's compression, and the use of logarithms, and, moreover, took this clumsy method of perpetuating their knowledge? Is it not far more probable that the architect simply determined that the length of the side should be 444 cubits, and accordingly measured out that distance?

"*Ex.* 3. Again, if instead of taking Colonel Vyse's measure of the base, we accept for that quantity the mean of half a dozen of the most trustworthy travellers' statements,—for instance, Vyse's, the French Academicians', Caviglia's, Wilkinson's, Lane's, and Davison's,—we have for the length of the side in English feet a number which expresses in millimètres the mean height of the barometer at Upsala. But will any one maintain that the dimensions of the pyramid were intended by its builders to have any reference whatever to that interesting constant?

"*Ex.* 4. If we multiply together one-tenth of the side of the pyramid's base, the length of the line joining the middle of the side and the apex (that is, the height of each one of the four isosceles triangles that compose the pyramid), and the modulus for the common logarithms, the result is 3420, the constant of lunar parallax, for which, in Burg's tables, the value 3420″·96 is given.

"*Ex.* 5. Lastly, if the side of the base (763·81) be divided by the hyperbolic logarithm of π (the circumference to diameter), and that quotient again by the ratio of the force of gravity at London to the force of gravity at the pyramid (lat. = 30°), (1·00188), the result is 666!"

4. On the best Arrangement for producing a Pure Spectrum on a Screen. By J. Clerk Maxwell, M.A., F.R.SS. L. & E.

In experiments on the spectrum, it is usual to employ a slit through which the light is admitted, a prism to analyse the light, and one or more lenses to bring the rays of each distinct kind to a distinct focus on the screen. The most perfect arrangement is that adopted by M. Kirchhoff, in which two achromatic lenses are used, one before and the other after the passage of the light through the prism, so that every pencil consists of parallel rays while passing through the prism.

But when the observer has not achromatic lenses at his command, or when, as in the case of the highly refrangible rays, or the rays of heat, he is restricted in the use of materials, it may still be useful to be able to place the lenses and prism in such a way as to bring the rays of all colours to their foci at approximately the same distance from the prism.

We shall first examine the effect of the prism in changing the convergency or divergency of the pencils passing through it, and then that of the lens, so that by combining the prism and the lens we may cause their defects to disappear.

When a pencil of light is refracted through a plane surface, its convergency or divergency is less in that medium which has the greatest refractive index; and this change is greater as the angle of incidence is greater, and also as the index of refraction is greater.

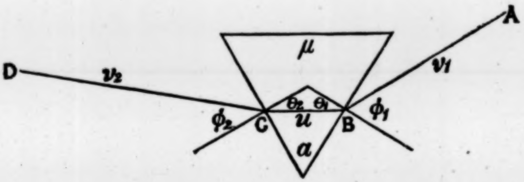
When the pencil passes through a prism its convergency or divergency will be diminished as it enters, and will be increased when it emerges, and the emergent pencil will be more or less convergent or divergent than the incident one, according as the angle of emergence is greater or less than that of incidence. This effect will increase with the difference of these angles and with the refractive index.

When the angle of incidence is equal to that of emergence the convergence of the pencil is unaltered, but since the more refrangible rays have the greatest angle of emergence, their convergency or divergency will be greater than that of the less refrangible rays.

This effect will be increased by making the angle of incidence less, and that of emergence greater; and it will be diminished by increasing the angle of incidence, that is, by turning the prism round its edge towards the slit. If the angle of the prism is not too great, the convergency or divergency of all the pencils may be made the same (approximately) by turning the prism in this way.

This correction, however, diminishes the separation of the colours. It is inapplicable to a prism of large angle, and it takes no account of the chromatic aberration of the lens. By altering the arrangement, the lens may be made to correct the prism. The effect of a convex lens is to increase the convergency, and to diminish the divergency, of every pencil; but the change is greatest on the most refrangible rays. The prism, except when its base is very much turned towards the slit, makes the highly refrangible rays more convergent or more divergent than the less refrangible rays, according as they were convergent or divergent originally. If the rays pass through the prism before they reach the lens, the pencils will be divergent at incidence, and the more refrangible will be most divergent at emergence. If they then fall on the lens, they will be more converged than the rest; so that by a proper arrangement all may be brought to their foci at approximately the same distance. If the violet rays come to their focus first, we must turn the base of the prism more towards the light, and *vice versa.*

We proceed to the numerical calculation of the proper arrangement.



To find the variation of position of the focus of light passed through a prism as dependent on the nature of the light.

Let μ be the index of refraction of the prism, a its angle, ϕ_1 and ϕ_2 the angles of incidence and emergence, θ_1 and θ_2 the angles of the ray within the prism with the normals to the first and second surfaces, δ the difference of these angles—then by geometry

$$\theta_1 + \theta_2 = . \quad a \qquad\qquad \theta_1 - \theta_2 = \quad \delta$$

and by the law of refraction,

$$\sin \phi_1 = \mu \sin \theta_1 \qquad\qquad \sin \phi_2 = \mu \sin \theta_2$$

ϕ_1 is constant, being the angle of incidence for all kinds of light, but the other angles vary with μ, so that

$$\frac{d\theta_1}{d\mu} = -\frac{\sin \theta_1}{\mu \cos \theta_1} \qquad \frac{d\theta_2}{d\mu} = \frac{\sin \theta_1}{\mu \cos \theta_1} \qquad \frac{d\phi_2}{d\mu} = \frac{\sin a}{\cos \theta_1 \cos \phi_2}$$

The last expression gives the dispersion, or breadth of the spectrum, and shows that it increases as the base of the prism is turned from the light.

As the slit is parallel to the edge of the prism, we have only to consider the primary foci of the pencils when we wish to render the image distinct.

Let v_1 be the distance of the focus of incident light from the prism, v_2 that of the emergent light, and u that within the prism, all measured to the right, then by the ordinary formula,

$$\frac{v_1}{\cos^2\phi_1} = \frac{u}{\mu\cos^2\theta_1} \qquad \frac{u}{\mu\cos^2\theta_2} = \frac{v_2}{\cos^2\phi_2}$$

$$v_1\cos^2\theta_1\cos^2\phi_2 = v_2\cos^2\theta_2\cos^2\phi_1 \;.$$

Taking the differential coefficient of the logarithms of these quantities,

$$\frac{1}{v_1}\frac{dv_1}{d\mu} - \frac{2\sin\theta_1}{\cos\theta_1}\frac{d\theta_1}{d\mu} - \frac{2\sin\phi_2}{\cos\phi_2}\frac{d\phi_2}{d\mu} = \frac{1}{v_2}\frac{dv_2}{d\mu} - \frac{2\sin\theta_2}{\cos\theta_2}\frac{d\theta_2}{d\mu}$$

$$\frac{1}{v_1}\frac{dv_1}{d\mu} - \frac{1}{v_2}\frac{dv_2}{d\mu} = \frac{2\sin\varphi_2\sin a}{\cos\theta_1\cos^2\varphi_2} - \frac{2\sin^2\theta_1}{\mu\cos^2\theta_1} - \frac{2\sin\theta_1\sin\theta_2}{\mu\cos\theta_1\cos\theta_2}$$

Substituting for these angles their values in terms of a and δ we find—

$$\frac{1}{v_1}\frac{dv_1}{d\mu} - \frac{1}{v_2}\frac{dv_2}{d\mu}$$

$$= \frac{4\sin a}{(\cos a + \cos\delta)^2} \cdot \frac{(\mu^2 - 1)\sin a - \sin\delta\,(1 + \mu^2\cos a - \delta)}{1 - \frac{1}{2}\mu^2\big(1 - \cos(a - \delta)\big)}$$

The quantity on the right of this equation is always positive, unless the value of δ exceeds that given by the equation

$$(\mu^2 - 1)\sin a = \sin\delta\,\big(1 + \mu^2\cos(a - \delta)\big)$$

If $\mu = 1{\cdot}5$ and $a = 60°$, then $\delta = 22°\,52'$, that is, the ray within the prism makes an angle of 11° 26′ with the base, which corresponds to an angle of incidence 82° 50′, or the incident ray is inclined 7° 10′ to the face of the prism.

If two lenses are used, of the same material with the prism, we may correct the defects of the prism without turning it so far from its position of least deviation.

Let a be the distance of the slit from the prism, and b that of the screen from the prism. Let f_1 be the focal length of the lens between the prism and slit, and f_2 that of the lens between the prism and the screen, then the condition of a flat image is

$$\frac{1}{v_1}\frac{dv_1}{d\mu} - \frac{1}{v_2}\frac{dv_2}{d\mu} = \frac{1}{\mu - 1}\left(\frac{v_1}{f_1} + \frac{v_2}{f_2}\right)$$

Let us first find the conditions of a flat spectrum $\delta = o$. When $\delta = o$, $v_1 = v_2$, and we obtain the conditions

$$\frac{1}{a}-\frac{1}{f_1}=\frac{1}{f_2}-\frac{1}{b}=\left(\frac{1}{a}+\frac{1}{b}\right)\frac{(1+\cos\alpha)^2\left(1-\frac{1}{2}\mu^2(1-\cos\alpha)\right)}{4(\mu-1)^2(\mu+1)\sin^2\alpha}=\left(\frac{1}{a}+\frac{1}{b}\right)c$$

from which f_1 and f_2 may be found in terms of known quantities.

$$\frac{1}{f}=\frac{1}{a}(1-c)-\frac{1}{b}c$$

$$\frac{1}{f_2}=\frac{1}{b}(1+c)+\frac{1}{a}c$$

When α, the angle of the prism, is 60°, then

$$c=\frac{3(4-\mu^2)}{16(\mu-1)^2(\mu+1)}$$

If we now make $\mu = 1{\cdot}5$ we find $c = {\cdot}5025$, and

$$\frac{1}{f_1}=\frac{{\cdot}4975}{a}-\frac{{\cdot}5025}{b}$$

$$\frac{1}{f_2}=\frac{1{\cdot}5025}{b}+\frac{{\cdot}5025}{a}$$

If $a = \frac{19}{21}b$, the first lens may be dispensed with, and the second lens will correct the defects of the prism.

If a is greater in proportion to b, a concave lens must be placed in front of the prism. The most convenient arrangement will be that in which the prism is placed in the position of least deviation, and the lens placed between the prism and the screen, while the distance from the slit to the prism is to that between the prism and the screen as $1-c$ is to c. For quartz, in which $\mu = 1{\cdot}584$ for the ordinary ray, $\frac{1}{c} = 2{\cdot}53$, so that the best arrangement is $a = 1{\cdot}53\,b$, or the lens should be placed on the side next the screen, and the distance from the slit which admits the light to the prism should be about one and a half times the distance from the prism to the screen.

The following Gentlemen were admitted Fellows of the Society:—

Rev. David Aitken, D.D.
Robert M. Ferguson, Ph.D.

Monday, 20*th January* 1868.

Professor KELLAND, Vice-President in the Chair.

The following Communications were read:—

1. Pyramidal Structures in Egypt and elsewhere; and the Objects of their Erection. By Sir J. Y. Simpson, Bart.

After considering the many proposed derivations of the word Pyramid, it was pointed out that the origin of the name suggested by the distinguished Egyptologist, Mr Birch, from two Coptic words, "*pouro*," "the king," and "*emahau*" or "*maha*," "tomb,"—the two in combination signifying "the king's tomb,"—was probably correct. "*Men*," in Coptic, signifies "monument," "memorial;" and "*pouro-men*," or "king's monument" may possibly also be the original form of the word. Various authors, as Pope, Pownall, Daniel Wilson, Burton, had long applied the term pyramid to the larger forms of conical and round sepulchral mounds, cairns, or barrows — such as are found in Ireland, Brittany, Orkney, &c., and in numerous districts of the New World as well as the Old; and which are all characterised by containing in their interior, chambers or cells, constructed usually of large stones, and with megalithic galleries leading into them. In these chambers (small in relation to the hills of stone or earth in which they were imbedded) were found the remains of sepulture, with stone weapons, ornaments, &c. The galleries and chambers were roofed, sometimes with flags laid quite flat, or placed abutting against each other; and occasionally with large stones arranged over the internal cells in the form of a horizontal arch or dome. In his travels to Madeira and the Mediterranean (1840), Sir W. Wilde details in interesting terms his visit to the pyramids of Egypt; and in describing the roof of the interior chambers of one of the pyramids at Sakkara,* he remarks on the analogy of its construction to the great barrow of Dowth in Ireland; and again, when writing—in his work on the Beauties of the Boyne (1849)—an account of the great old barrows of Dowth, New Grange, &c., placed on its banks above Drogheda, he describes at some length the last of these mounds (New Grange), stating that it "consists" of an enormous cairn or "hill of small stones, calculated at 180,000 tons weight, occupying the summit of one of the natural undulating slopes which enclose the valley of the Boyne upon the north. It is said to cover nearly two acres, and is 400 paces in circumference, and now about 80 feet higher than the adjoining natural surface. Various excavations (he adds) made into its sides and upon its summit, at different times, in order to supply materials for building and road-making, have assisted to lessen its original height, and also to destroy the beauty of its outline." Like the other analogous mounds and pyramids placed there and elsewhere, New Grange has a long megalithic gallery, of above 60 feet in length, leading inward into three dome-shaped chambers or crypts. After describing minutely, and with a master-hand, the construction of these interior parts, and the carvings of circles, spirals, &c., upon the enormous stones of which they are built, Sir William Wilde goes on to observe:—"We believe with most modern investigators into such subjects, that it was a tomb, or great sepulchral Pyramid, similar in every respect to those now standing by the banks of the Nile, from Dashour to Gizeh, each consisting of a great central chamber containing one or more sarcophagi, entered by a long stone-covered passage. The external aperture was concealed, and the whole covered with a great mound of stones or earth in a conical form. The early Egyptians, and the Mexicans also, possessing greater art and better tools than the primitive Irish, carved, smoothed, and cemented their great pyramids; *but the type and purpose is all the same*. . . . How far anterior to the Christian era its date should be placed, would be a matter of speculation; it may be of an age coeval, or even anterior, to its brethren on the Nile." Other pyramidal barrows at Maeshowe, Gavr Inis, &c., were referred to and illustrated; showing that a gigantic sepulchral cairn was in its mass an unbuilt pyramid, or, in other words, that a pyramid was a built cairn.

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* Sir J. Gardner Wilkinson thinks that the pyramids of Sakkara are probably older than the other groups of these structures. as those of Gizeh or the Great Pyramid.—See Rawlinson's *Herodotus*, vol. ii. chapter viii.

Sepulchral Character, &c., of the Egyptian Pyramids.

All authors, from the father of history downwards, have generally agreed in considering the pyramids of Egypt as magnificent and regal sepulchres; and the sarcophagi, &c., of the dead have been found in them when first opened for the purposes of plunder or curiosity. The pyramidal sepulchral mounds on the banks of the Boyne were opened and rifled in the ninth century by the invading Dane, as told in different old Irish annals; and the Great Pyramid of Gizeh, &c., is said to have been broken into and harried in the same century by the Arabian Caliph, El Mamoon. This, the largest and most gigantic of the many pyramids of Egypt, had been calculated by Major Forlong, Asso. Inst. C. Engrs., as a structure which in the East would cost about L.1,000,000. Over India and the East generally, enormous sums had often been expended on royal sepulchres. The Taj Mahal of Agra, built by the Emperor Shah Jahan for his favourite queen, cost perhaps double or triple this money; and yet it formed only a portion of an intended larger mausoleum which he expected to rear for himself. The Great Pyramid contains in its interior, and directly over the King's Chamber, five entresols or "chambers of construction," as they have been termed, intended apparently to take off the enormous weight of masonry from the cross stones forming the roof of the King's Chamber itself. These entresols are chambers, small and unpolished, and never intended to be opened. But in the uppermost of them, opened by Colonel Howard Vyse, a most interesting discovery was made about thirty years ago. The surfaces of some of the stones were found painted over in red ochre or paint with rudish hieroglyphics—being quarry marks written on the stones 4000 years ago, and hence, perhaps, forming the oldest preserved writing in the world. Among these accidental hieroglyphics Mr Birch discovered two royal ovals, viz., Shufu (the Cheops of Herodotus) and Nu Shufu—"a brother" (writes Professor Smyth) "of Shufu, also a king and a co-regent with him." Most, if not all, of the other pyramids are believed to have been erected by individual kings during their individual or separate reigns. If these hieroglyphics proved that two kings were connected with the building of the Great Pyramid, that circumstance would perhaps account for its size and the duplicity and position of its sepulchral chambers.* The Great Pyramid is now totally deprived of the external polished limestone coating which covered it at the time of Herodotus' visit, some twenty-two centuries ago; and "now (writes Mr Smyth) is so injured as to be in the eyes of some passing travellers little better than a heap of stones." But all the internal built core of the magnificent structure remains, and contains in its interior (beside a rock chamber below) two higher built chambers or crypts above—the so-called King's Chamber and Queen's Chamber—with galleries and apartments leading to them. The walls of these galleries and upper chambers are built with granite and limestone masonry of a highly-finished character. The pyramid standing next the Great Pyramid, and nearly of equal size, is said by Herodotus to have been raised by the brother of Cheops. The other pyramids at Gizeh are usually regarded as later in date. But the exact era of the reign or reigns of their builders has not as yet been determined, in consequence of the break made in Egyptian chronology by the invasion of the Shepherd Kings.

In their mode of building, the pyramids of Gizeh, &c., are all similar. "There is nothing" (observes Professor Smyth) "in the stone-upon-stone composition of the Great Pyramid which speaks of the mere building problem to be solved there, as being of a different character or requiring inventions by man of any absolutely higher order, than elsewhere." But the Great Pyramid has been imagined to contain some hidden symbols and meanings. For "it is the manner of the Pyramid" (according to Professor Smyth) "not to wear its most vital truths in prominent outside positions."

Alleged Metrological Object of the Great Pyramid.

By several authorities the largest† of the group of pyramids at Gizeh, the "Great Pyramid," as it is often termed, has been maintained—and particularly of late by Gabb, Jomard, Taylor, and Professor Smyth—not to be a royal mausoleum, but to be a marvellous metrological monument, built some forty centuries ago, as "a necessarily material centre," to hold and contain within it, and in its structure, material standards, "in a practicable and reliable shape," "down to the ends of the world," as measures of length, capacity, weight, &c., for men and nations for all time—"a monument" (in the language of Professor Smyth) "devoted to weights and measures, not so much as a place of frequent reference for them, but one where the original standards were to be preserved for some thousands of years, safe from the vicissitudes of empires and the decay of nations." Messrs Taylor and Smyth further hold that this Great Pyramid was built for these purposes of mensuration, under Divine inspiration—the standards being, through superhuman origination and guidance, made and protected by it till they came to be understood and interpreted in these latter times. For, observes Professor Smyth, "the Great Pyramid was a sealed book to all the world *until* this present day, when modern science, aided in part by the dilapidation of the building and the structural features thereby opened up—has at length been able to assign the chief interpretations." Professor Smyth has, in his remarkable devotedness and enthusiasm, lately measured most of the principal points in the Great Pyramid; and for the great zeal, labour, and ability which he has displayed in this self-imposed mission, the Society have very properly and justly bestowed upon him the Keith Medal. But the exactitude of the measures does not necessarily imply exactitude in the reasoning upon them; and on what grounds can it be possibly regarded as a metrological monument and not a sepulchre, is legitimately the subject of our present inquiry.

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* Mr Birch, however—and it is impossible to cite a higher authority in such a question—holds the cartouches of Shufu and Nu Shufu to refer only to one personage—namely, the Cheops of Herodotus; and, believing with Mr Wilde and Professor Lepsius, that the pyramids were as royal sepulchres built and methodically extended and enlarged as the reigns of their intended occupants lengthened out, he ascribes the unusual size of the Great Pyramid to the unusual length—as testified by Manetho, &c.—of the reign of Cheops; the erection of a sepulchral chamber in its built portion being, perhaps, in consequence of some ascertained deficiency in the rock chamber or gallery below.

† The Mexican Pyramid of Cholula has a base of more than 1420 feet, and is hence about twice the length of the basis of the Great Pyramid of Gizeh. See Prescott's *Conquest of Mexico*, book iii. chap. i., and book v. chap. iv.

Mr Taylor ascribes to Noah the original idea of the metrological structure of the Great Pyramid. "To Noah" (observes Mr Taylor) "we must ascribe the original idea, the presiding mind, and the benevolent purpose. He who built the Ark was, of all men, the most competent to direct the building of the Great Pyramid. He was born 600 years before the Flood, and lived 350 years after that event, dying in the year 1998 B.C. Supposing the pyramids were commenced in 2160 B.C. (that is 4000 years ago), *they* were founded 168 years before the death of Noah. We are told "(Mr

Taylor continues) "that Noah was a 'preacher of righteousness,' but nothing could more illustrate this character of a preacher of righteousness after the Flood than that he should be the first to publish a system of weights and measures for the use of all mankind, based upon the measure of the earth." Professor Smyth, computing by another chronology, rejects the presence of Noah, and makes a shepherd—Philithion, alluded to by Herodotus*—the presiding and directing genius of the structure, holding him to be a Cushite skilled in building, and under whose inspired direction the pyramid rose, containing within it miraculous measures and standards of capacity, weight, length, heat, &c.

THE COFFER IN THE KING'S CHAMBER IN THE GREAT PYRAMID AN ALLEGED STANDARD FOR MEASURES OF CAPACITY.

A granite coffer, stone box, or sarcophagus standing in that interior cell of the pyramid called the King's Chamber, is held by Messrs Taylor and Smyth to have been hewn out and placed there as a measure of capacity for the world, so that the ancient Hebrew, Grecian, and Roman measures of capacity on the one hand, and our modern Anglo-Saxon on the other, are all derived directly or indirectly from the parent measurements of this granite vessel.

Various extracts† were read from the works of these authors, showing that, in their belief, the great object for which the whole pyramid was created, was the preservation of this coffer as a standard of measures. Further published accounts from Mr Taylor and from Mr Smyth's first work were adduced, averring that the coffer represented without and within a rectangular figure of mathematical form and "exquisite geometric truth," perfectly level on its surface, and highly polished. "The chest or coffer in the Great Pyramid" (writes Mr Taylor in 1859,) "is so shaped as to be in every part rectangular from side to side, and from end to end, and the bottom is also cut at right angles to the sides and end, and made perfectly level." "The coffer," said Professor Smyth in 1864, "exhibits to us a standard measure of 4000 years ago, with the tenacity and hardness of its substance unimpaired, and the polish and evenness of its surface untouched by nature through all that length of time."

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* Herodotus states that the Egyptians detested the memories of the kings who built the two larger Pyramids, viz., Cheops and Cephren; and hence, he adds, "they commonly call the Pyramids after Philithion, a shepherd, who at that time fed his flocks about the place." They thus called the second, as well as the Great Pyramid, after him (iii. § 128).

† The extracts within inverted commas, here and in other parts, are from—(1.) Mr John Taylor's work entitled "The Great Pyramid—Why was it Built, and Who Built it?" London, 1859; and (2.) Professor Smyth's work, "Our Inheritance in the Great Pyramid," Edinburgh, 1864; (3.) his later three-volume work, "Life and Work at the Great Pyramid," Edinburgh, 1867; and (4.) "Recent Measures at the Great Pyramid" in the Transactions of the Royal Society of Edinburgh for 1865–66.

But later inquiries and observations upset entirely all these notions and strong averments in regard to the coffer. For—

(1.) *The Coffer, though an alleged actual standard of capacity-measure, has yet been found difficult or impossible to measure.*—In his first work, "Our Inheritance in the Great Pyramid," Professor Smyth had cited the measurements of it, made and published by twenty-five different observers, several of whom had gone about the matter with great mathematical accuracy. Though imagined to be a great standard of measure, yet all these twenty-five, as Professor Smyth owned, varied from each other in their accounts of this imaginary standard in "every element of length, breadth, and depth, both inside and outside." Professor Smyth has latterly measured it himself, and this twenty-sixth measurement varies again from all the preceding twenty-five. Surely a measure of capacity should be measurable; that ought to be its most unquestionable quality; but this imagined standard has proved virtually unmeasurable—in so far at least that its twenty-six different and skilled measurers all differ from each other in respect to its dimensions. Still, says Professor Smyth, "this affair of the coffer's precise size is *the* question of questions."

(2.) *Discordance between its actual and its theoretical measure.*—Professor Smyth holds that *theoretically* its capacity ought to be 71,250 "pyramidal" cubic inches, for that cubic size would make it the exact measure for a chaldron, or practically the vessel would then contain exactly four quarters of wheat, &c. Yet Professor Smyth himself found it some 60 cubic inches less than this; while also the measurements of Professor Greaves, one of the most accurate measurers of all, make it 250 cubic inches, and those of

Dr Whitman 14,000 *below* this professed standard. On the other hand, the measurements of Colonel Howard Vyse make it more than 100, those of Dr Wilson more than 500, and those of the French academicians who accompanied the Napoleonic expedition to Egypt, about 6000 cubic inches *above* the theoretical size which Professor Smyth has latterly fixed on.

(3.) *Its theoretical measure varied.*—The *actual* measure of the coffer has varied in the hands of all its twenty-six measurers. But even its *theoretical* measure is varied also, for the size which the old coffer really *ought* to have as "a grand capacity standard," is strangely enough not a determined quantity. In his last work (1867), Professor Smyth declares, as just stated ,its proper theoretical cubic capacity to be 71,250 pyramidal cubic inches. But in his first work (1864), he declared something different, for "we *elect*," says he, "to take 70,970·2 English cubic inches (or 70,900 pyramidal cubic inches) as the true, because the theoretically *proved* contents of the porphyry coffer, and therefore accept these numbers as giving the cubic size of the grand *standard* measure of capacity in the Great Pyramid." But again, Mr John Taylor, who, previously to Professor Smyth, was the great advocate of the coffer being a marvellous standard of capacity measure for all nations, ancient and modern, declares its measure to be neither the above quantities, but 71,328 cubic inches, or a cube of the ancient cubit of Karnak.* A vessel cannot be a measure of capacity whose own standard theoretical size is thus declared to vary somewhat every few years by those very men who maintain that it is a standard. But whether its capacity is 71,250, or 70,970, or 71,328, it is quite equally held up by Messrs Taylor and Smyth that the Sacred Laver of the Iraelites, and the Molten Sea of the Scriptures, "also conform and correspond to its (yet undetermined) standard with *all* conceivable practical exactness;" though the standard of capacity to which they thus "conform and correspond" is itself a size or standard which has not been yet fixed with any exactness. Professor Smyth, in speaking of the calculations and theoretical dimensions of this coffer—as published by Mr Jopling, a believer in its wonderful standard character—critically observes, "Some very astonishing results were brought out in the play of arithmetical numerations."

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* "Its contents," says Mr Taylor (p. 299), "are equal in cubic inches to the cube of 41,472 inches—the cubit of Karnak—viz., to 71,328 cubic inches." Elsewhere (p. 304) he states—"The Pyramid coffer contains 256 gallons of wheat;"—"It also contains 256 gallons of water, &c."

(4.) *The dilapidation of the Coffer.*—Thirty years ago this stone coffer was pointed out and indeed delineated by Mr Perring as "*not* particularly well polished," and "chipped and broken at the edges." Professor Smyth, in his late travels to Egypt, states that he found every possible line and edge of it chipped away with large chips along the top, both inside and outside, "chip upon chip, wofully spoiling the original figure; along all the corners of the upright sides too, and even along every corner of the bottom, while the upper south-eastern corner of the whole vessel is positively broken away to a depth and breadth of nearly a third of the whole." Yet this stone vessel is professed to be the permanent miraculous standard of capacity measure for the world, and that it might serve this purpose it would be, according to Mr Taylor, "formed of one block of the hardest kind of material, such as porphyry or granite, *in order* that it might *not* fall into decay;" for "in this porphyry coffer we have (writes Professor Smyth in 1864) the very closing end and aim of the whole pyramid."

(5.) *Alleged mathematical form of the Coffer erroneous.*—But in regard to the coffer as an exquisite and marvellous standard of capacity to be revealed in these latter times, worse facts than these are divulged by the tables, &c., of measurements which Professor Smyth has recently published of this stone vessel or chest. His published measurements show that it is not at all a vessel, as was averred a few years ago, of pure mathematical form; for externally it is in length an inch greater on one side than another; in breadth, half-an-inch broader at one point than at some other point; its bottom at some points is nearly a whole inch thicker than it is at some other parts; and in thickness its sides vary in some points about a quarter of an inch near the top. "But," Professor Smyth adds, "if calipered lower down, it is extremely probable that a *notably* different thickness would have been found there," though it does not appear why they were not thus calipered. Further, externally "all the sides" (says Professor Smyth) "were slightly hollow, excepting the east side;" and the "two external ends" also show some "concavity" in form. "The workmanship" (he elsewhere describes) "of the inside is in advance of the outside, but yet *not* perfect." For internally there is a convergence at the bottom towards the centre; both in length and in breadth the interior differs about half-an-inch at one point from another point; the "extreme points (also) of the corners of the bottom not being perfectly worked out to the intersection of the general planes of the entire sides;" and thus its cavity seems really of a form utterly unmeasurable in a correct way by mere lineal measurement—the only measure yet attempted. If it were an object of the slightest moment, perhaps liquid measurements would be more successful.

(6.) *Coffer cut with ledges and catch-holes for a lid, like other sarcophagi.*—More damaging details still remain in relation to the coffer as "a grand standard measure of capacity," and proving that its object or function was very different. In his first work Prof. Smyth describes the coffer as showing no "symptoms" whatever of grooves or catchpins or other fastenings for a lid. "More modern accounts," he re-observes, "have been further precise in describing the smooth and geometrical finish of the upper part of the coffer's sides, *without any* of those grooves, dovetails, or steady-pin-holes which have been found elsewhere in true polished sarcophagi, where the firm fastening of the lid is one of the most essential features of the whole business." Mr Perring, however, delineated the catchpin-holes for a lid in the coffer thirty years ago.* On his late visit to it Prof. Smyth found its western side lowered down in its whole extent to nearly an inch and three-quarters (or more exactly, 1·72 inch), and ledges cut out around the interior of the other sides at the same height. Should we measure on this western side from this actual ledge brim, or from the imaginary higher brim? If reckoned as the true brim, "this ledge" (according to Professor Smyth) would "take away near 4000 inches from the cubic capacity of the vessel." Besides, he found three holes cut on the top of the coffer's lowered western side, as in all the other Egyptian sarcophagi, where they are used with the ledge to admit and lock the lids of such stone chests.* In other words, it presents the usual ledge and apparatus pertaining to Egyptian stone sarcophagi, and served as such.

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* See Plate III. fig. 1, in his great folio work on the "Pyramids of Gizeh from Actual Survey and Admeasurement," Lond. 1839. "The sarcophagus is," he remarks, "of granite, not particularly well polished; at present it is chipped and broken at the edges. There are not any remains of the lid, *which was, however,* fitted on in the same manner as those of the other pyramids."

7. *Sepulchral contents of coffer when first discovered.*—When, about a thousand years ago, the Caliph Al Mamoon tunnelled into the interior of the pyramid, he detected by the accidental falling, it is said, of a granite portcullis, the passage to the King's Chamber, shut up from the building of the pyramid to that time. "Then" (to quote the words of Prof. Smyth) "the treasures of the pyramid, sealed up almost from the days of Noah, and undesecrated by mortal eye for 3000 years, lay full in their grasp before them." The Arabian historian Ibn Abd Alhokim states that on this occasion they found in the pyramid, "towards the top, a chamber [now the so-called King's Chamber] with an hollow stone [or coffer] in which there was a statue like a man, and within it a man upon whom was a breastplate of gold set with jewels; upon this breastplate was a sword of inestimable price, and at his head a carbuncle of the bigness of an egg, shining like the light of the day; and upon him were characters writ with a pen, which no man understood"†—a description stating, down to the so-called "statue," mummy-case, or cartonage, and the hieroglyphics upon the cerecloth, the arrangements now well known to belong to the higher class of Egyptian mummies.

* "The western side," observes Professor Smyth, "of the coffer is, through almost its entire length, rather lower than the other three, and these have *grooves* inside, or the remains of grooves once cut into them, about an inch or two below their summits and on a level with the western edge; *in fact*, to admit a sliding sarcophagus cover or lid; and there were the remains of three fixing pin-holes on the western side for fastening such cover into its place." (Vol. i. p. 85.)

† See Greaves' Works, vol. i. p. 61. Colonel Vyse adduces other Arabian authors who allude to this discovery of a body with golden armour, &c., &c., in the sarcophagus of the King's Chamber. He cites Alkaisi as testifying that "he himself saw the case (the cartonage or mummy-case) from which the body had been taken, and that it stood at the door of the King's Palace at Cairo, in the year 511" A.H. (See "The Pyramids of Gezah," vol. ii. p. 334. And also to the same effect Abon Szalt, p. 357.) "It may be remarked," observes Colonel Vyse, "that the Arabian authors have given the same accounts of the pyramids, with little or no variation, for above a thousand years" (vol. ii. p. 328).

In short (to quote the words of Prof. Smyth), "that wonder within a wonder of the Great Pyramid—viz., the porphyry coffer," "this vessel of exquisite meaning" and of "far-reaching characteristics," mathematically formed under alleged Divine inspiration as a measure of capacity (and, according to M. Jomard, of length also) for all men and all nations, for all time, and particularly for these latter profane times, is, in simple truth, nothing more and nothing less than—an old and somewhat misshapen stone coffin.

STANDARD OF LINEAR MEASURE IN THE GREAT PYRAMID.

The standard in the Great Pyramid, according to Messrs Taylor and Smyth, for *linear* measurements, is the length of the base line or lines of the pyramid. This, Professor Smyth states, is "the function proper of the pyramid's base." It is professed also that in this base line there has been found a new mythical inch—one-thousandth of an inch longer than the British standard inch; and Professor Smyth has attempted to show that the status of the kingdoms of Europe in the general and moral world may be measured in accordance with their present deviation or conformity to this mythical pyramidal standard in their modes of national measurement. "For the linear measure" (says Prof. Smyth), "of the base line of this colossal monument, viewed in the light of the philosophical connection between time and space, has yielded a standard measure of length which is more admirably and learnedly earth-commensurable than anything which has ever yet entered into the mind of man to conceive, even up to the last discovery in modern metrological science, whether in England, France, or Germany."

The engineers and mathematicians of different countries have repeatedly measured a meridian arc of a degree of sixty miles in order to use it as a standard for linear divisions. As part of their standard, they measure off sixty miles of the irregular earth-surface of a kingdom with almost perfect mathematical exactitude. Professor Smyth holds that the basis side of the pyramid has been laid down by Divine authority as such a guiding standard measure.

What, then, is the exact length of one of its basis lines? The sides of the pyramid have been measured by many different measurers. Linear standards have, says Professor Smyth, "been already looked for by many and many an author on the sides of the base of the Great Pyramid, even before they knew that the terminal points of those magnificent base lines had been carefully marked in the solid rock of the hill by the socket-holes of the builders." But—as in the case of the cubic capacity of the coffer these measurers sadly disagree with each other in their measurements, which, in fact, but vary from some 7500 or 8000 inches to 9000 and upwards. Thus, for example, Strabo makes it under 600 Grecian feet, or under 7500 English inches; Dr Shawe makes it 8040 inches; Shelton makes it 8184 inches; Greaves, 8316; Davison, 8952; Caviglia, 9072; the French academicians, 9163; Dr Perry, 9360, &c., &c.

At the time at which Professor Smyth was living at the pyramid, Mr Inglis, of Glasgow, visited it, and, for correct measurement, laid bare for the first time the four corner sockets. Mr Inglis' measurements not only differed from all the other measurements of "one-side" base lines made before him, but he makes the four sides differ from each other; one of them—namely, the north side—being longer than the other three. Strangely, Professor Smyth, though in Egypt for the purpose of measuring the different parts of the pyramid—and holding that its base line ought to be our grand standard of measure,—and further holding that the base line could only be accurately ascertained by measuring from socket to socket—never attempted that linear measurement himself after the sockets were cleared. These four corner sockets were never exposed before in historic times; and it may be very long before an opportunity of seeing and using them again shall ever be afforded to any other measurers.

Before the corner sockets were exposed, Professor Smyth attempted to measure the bases, and made each side of the present masonry courses "between 8900 and 9000 inches in length," or (to use his own word) "*about*" 8950 inches for the mean length of one of the four sides of the base; exclusive of the ancient casing and backing stones—which last Colonel Howard Vyse found and measured to be precisely 108 inches on each side, or 216 on both sides. These 216 inches, added to Professor Smyth's measure of "about" 8950 inches, makes one side 9166 inches. But Professor Smyth has "elected" (to use his own expression) not to take the mathematically exact measure of the casing stones as given by Colonel Vyse and Mr Perring, who alone ever saw them and measured them (for they were destroyed shortly after their discovery in 1837), but to take them, without any adequate reason and contrary to their mathematical measurement, as equal only to 202 inches, and hence "accept 9152 inches as the original length of one side of the base of the finished pyramid." He deems, however, this "determination" not to be so much depended upon as the measurements made from socket to socket.

The mean of the only four series of such socket or casing stone measures as have been recorded hitherto by Jomard (9163), Vyse (9168), Mahmoud Bey (9162), and Inglis (9110), amounts to nearly 9150. The first three of these observers were only able to measure the north side of the pyramid. Mr Inglis measured all the four sides, and made them respectively 9120, 9114, 9102, and 9102, making a difference of 18 inches between the shortest and longest. Professor Smyth thinks the measures of Mr Inglis as on the whole probably too small, and he takes two of them, 9114 and 9102—(but, strangely, not the largest, 9120)—as data, and strikes a new number out of these two and out of the three previous measures of Jomard, Vyse, and Mahmoud Bey, from these five quantities making a calculation of "means," and electing 9142 as the proper measure of the basis line of the pyramid—(which exact measure certainly none of its many measurers ever yet found it to be); and upon this *foundation*, "derived" (to use his own words) "from the best modern measures yet made," he proceeds to reason on, "as the happy, useful, and perfect representation of 9142," and the great standard for linear measure revealed to man in the Great Pyramid. Surely it is a very strange standard of linear measure that can only be thus elicited and developed—not by direct measurement, but by indirect logic, and regarding the exact and precise length of which there is as yet no kind of reliable and accurate certainty.

Lately, Sir Henry James has shown that the length of one of the sides of the pyramid base, with the casing stones added, as measured by Colonel H. Vyse—viz., 9168 inches—is precisely 360 derahs, or land cubits of Egypt; the derah being an ancient land measure still in use, of the length of nearly 25½ British inches, or, more correctly, of 25·488 inches; and he has pointed out that in the construction of the body of the Great Pyramid, the architect built 10 feet or 10 cubits of horizontal length for every 9 feet or 9 cubits of vertical height; while in the construction of the various galleries the proportion was adhered to of horizontal length to vertical height as 9 to 4,—rules which would greatly simplify the building of such a structure. The Egyptian derah of 25·48 inches is practically one-fourth more in length than the old Egyptian cubit of Memphis of 20·7 inches. Long ago Sir Isaac Newton showed from Professor Greaves' measurements of the chambers, galleries, &c., that the Memphis cubit of 20·7 British inches was apparently the *working* cubit of the masons in constructing the pyramid—an opinion so far admitted more lately by both Messrs Taylor and Smyth; "the length" (says Professor Smyth) "of the cubit employed by the masons engaged in the Great Pyramid building, or that of the ancient city of Memphis," being 20·73 British inches.* According to Mr Inglis' late measurement of the four bases of the pyramid, after its four corner sockets were exposed, the length of each base line was possibly 440 Memphis cubits, or 9108 English inches; or, if the greater length of Jomard and Vyse be held nearer the truth, 442 Memphis cubits, or 9150 British inches.

But Professor Smyth tries to show that (1.) if 9142 be granted him as the possible base line of the pyramid; and (2.) if 25 inches be allowed to be the length of the "Sacred Cubit," as revealed to the Israelites (and as revealed in the pyramid), then the base line might be found very near a multiple of this cubit by the days of the year,† or by 365·25; for these two numbers multiplied together amount to 9131 pyramidal inches, or 9140 British inches—the British inch being held, as already stated, to be 1000th less than the pyramidal inch. Was, however, the "Sacred Cubit"—upon whose alleged length of 25 "pyramidal" inches this idea is entirely built—really a measure of this length? In this matter—the most important and vital of all for his whole linear hypothesis—Professor Smyth seems to have fallen into an error which upsets all the calculations and inferences founded by him upon it.

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* Yet this, the Memphian cubit, "need not" (somewhat mysteriously adds Professor Smyth), "and actually is not, by any means the same as the cubit *typified* in the more concealed and *symbolised* metrological system of the Great Pyramid."

† Godfrey Higgins, in his work on "The Celtic Druids," shows how, among the ancients, superstitions connected with numbers, as the days of the year or the figures 365, have played a prominent part. "Amongst the ancients" (says he) "there was no end of the superstitious and trifling play upon the nature and value of numbers. The first men of antiquity indulged themselves in these fooleries" (p. 244). Mr Higgins points out that the old Welsh or British word for Stonehenge, namely Emrys, signifies, according to Davies, 365; as do the words Mithra, Neilos, &c.; that certain collections of the old Druidic stones at Abary may be made to count 365; that "the famous Abraxas only meant the solar period of 365 days, or the sun," &c. "It was all judicial astrology. . . . It comes (adds Mr Higgins) from the Druids."

Sir Isaac Newton, in his remarkable "Dissertation upon the Sacred Cubit of the Jews," long ago came to the conclusion that it measured 24·82 British inches. In this way it was one-fifth longer than the cubit of Memphis, viz., 20·7 inches, as previously deduced by him from Greaves' measurements of the King's Chamber and other parts of the interior of the Great Pyramid. Before drawing his final inference as to the Sacred Cubit being 24·82 inches, and as so many steps conducting to that inference, Sir Isaac shows that the Sacred Cubit was some measurement intermediate between a long and moderate human step or pace, between the third of the length of the body of a tall and short man, &c., &c. Professor Smyth has collected eight of the measurements thus adduced by Newton as "methods of approach" to circumscribe the length of the Sacred Cubit; and adding to these data Sir Isaac's deduction of the actual length of the Sacred Cubit as a ninth quantity, he enters the whole nine in a table as follows:—

Professor Smyth's Table of Newton's Inquiry regarding the Sacred Cubit.

First,	between	23·28 and	27·94	British inches.
Second,	„	23·3	27·9	„
Third,	„	24·80	25·02	„
Fourth,	„	24·91	25·68	„
And Fifth, somewhere near 24·82.				

"The mean of all which numbers" (Professor Smyth remarks) "amounts to 25·07 British inches. The Sacred Cubit, then, of the Hebrews" (he adds) "in the time of Moses—*according to Sir Isaac Newton*—was equal to 25·07 British inches, with a probable error of ± ·1."

But—"*according* to Sir Isaac Newton"—the Sacred Cubit of the

Jews was *not* 25·07, as Professor Smyth makes him state, but 24·82 British inches (or 25 unciæ of the Roman foot, and $\frac{6}{10}$ of an uncia), as Sir Isaac himself more than once deliberately infers in his Dissertation. Besides, in such inquiries is it not illogical to attempt to draw mathematical deductions by these calculations of "means," and especially by using the ninth quantity in the table —viz., Sir Isaac's own deliberate deduction regarding the actual length of the Sacred Cubit—as one of the nine quantities from which that length was to be deduced by the equivocal process of means? Errors, however, of a still more serious kind exist. The "mean" of the nine quantities in Professor Smyth's table is, he infers, 25·07 inches; and hence he avows that this, or near this figure, is the length of the Sacred Cubit. But the real mean of these nine quantities is not 25·07, but 25·29—a number in such a testing question as this of a very different value. For the days of the year (365·25) when multiplied by this—the true mean of these nine quantities—would make the base line of the pyramid 9237 inches, instead of Professor Smyth's theoretical number of 9142 inches—a difference altogether overturning all his inferences and calculations thereanent.

The incidentally erroneous summation which Professor Smyth makes of the nine quantities in his table, as amounting to 25·07, he declares (to use his own strong words) as a "*really glorious consummation* for the geodesical science of the present day to have brought to light;" for he avers this length of 25·07—which he forthwith elects to alter to 25·025 British inches—being "practically the sacred Hebrew cubit, is *exactly* one ten-millionth (1–10,000,000th) of the earth's semi-axis of rotation; and *that* is the very best mode of reference to the earth-ball as a whole, for a linear standard through all time, that the highest science of the existing age of the world has yet struck out, or can imagine. In a word, the Sacred Cubit, *thus* realised, forms an instance of the most advanced and perfected human science supporting the truest, purest, and most ancient religion; while a linear standard which the chosen people in the earlier ages of the world were merely told by maxim to look on as *sacred*, compared with other cubits of other lengths, is proved by the progress of human learning in the latter ages of time, to have had, and still to have, a philosophical merit about it which no men or nations at the time it was first produced, or within several thousand years thereof, could have possibly thought of for themselves." Besides, adds he elsewhere, "an *extraordinary* (sic) convenient length, too, for man to handle and use in the common affairs of life is the one ten-millionth of the earth's semi-axis of rotation when it comes to be realised, for it is extremely close to the ordinary human arm, or to the ordinary human pace in walking with a purpose to measure."

Of course, all these inferences and reasonings, regarding the Sacred Cubit being an exact segment of the polar axis, disappear when we find Sir Isaac Newton's length of the sacred cubit is not, as Professor Smyth elects it to be, 25·025 British inches; nor 25·07 as he calculated it to be from the mean of the nine quantities in his table; nor 25·29, as is the actual mean of these nine quantities; but, "*according* to Sir Isaac Newton's" own reiterated statement and conclusion, 24·82. A Sacred Cubit, according to Sir Isaac Newton's admeasurements of it of 24·82 inches, would not, by hundreds of cubits, be one ten-millionth of the measure of the semi-polar axis of the earth; provided the polar axis be, as Professor Smyth elects it to be, 500,500,000 British inches.

AXIS OF THE EARTH AS A STANDARD OF MEASURE.

The standards of measure in most modern civilised countries are, as is well known, referred to divisions of arcs of the meridian, measured off upon different points of the surface of the earth. These measures of arcs of the meridian, as measurements of a known and selected portion of the surface of the spheroidal globe of the earth, have, more or less, fixed mathematical relations with the axis of the earth; as the circumference of a sphere has an exact mathematical ratio to its diameter. The difference in length of arcs of the meridian at different parts of the earth's surface, in consequence of the irregular spheroidal form of the globe of the earth, has led to the idea that the polar diameter or axis of the earth would form a more perfect and more universal standard than measurements of the surface of the earth. In the last century, Cassini* and Callet† proposed, on these grounds, that the polar axis of the earth should be taken as the standard of measure. Without having noticed these propositions of Cassini and Callet, Professor Smyth adopts the same idea, and avers that 4000 years ago it had been adopted and used also by the builders of the Great Pyramid, who laid out and measured off the basis of the pyramid as a multiple by the days of the year of the Sacred Cubit, and hence of the pyramidal cubit; while the Sacred or Pyramidal Cubit were both the results of superhuman or divine knowledge, and were both, or each, one ten-millionth of the semi-polar axis of the earth. We have already seen, however, that the Sacred Cubit, *according* to Sir Isaac Newton, is not a multiple by the days of the year of the base line of the Great Pyramid, and is not one twenty-millionth of the polar axis of the earth, when that polar axis is laid down as measuring, according to the numbers elected by Professor Smyth, 500,500,000 British inches.

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* "Traite de la Grandeur et de la Figure de la Terre." Amsterdam edition (1723), p. 195.

† "Tables Portatives de Logarithmes." Paris, 1795, p. 100.

But is there any valid reason whatever for fixing and determining, as an ascertained mathematical fact, the polar axis of the earth to be this very precise and exact measure, with its formidable tail of nothings? None, except the supposed requirements or necessities of Professor Smyth's pyramid metrological theory. The latest and most exact measurements are acknowledged to be those of Captain Clarke, who, on the doctrine of the earth being a spheroid of revolution, computes the polar axis to be 500,522,804 British inches; or 500,482,296 British inches, calculating it from the results of all the known arcs of meridian measures. If we grant that the Sacred Cubit could be allowed to be exactly 25·025 inches, which Sir Isaac Newton found it not to be; and if we grant that the polar axis is exactly 500,500,000 British inches, which Captain Clarke did not find it to be; then, certainly, as shown by Professor Smyth, there would be 20,000,000 of these supposititious pyramidal cubits, or 500,000,000 of the supposititious pyramidal inches in this supposititious polar axis of the earth. "In so far, then" (writes Professor Smyth), "we have in the 5, with the many 0's that follow, a pyramidally commensurable and symbolically appropriate unit for the earth's axis of rotation." But such adjustments have been made with as great apparent exactitude when entirely different earth-axes and quantities were taken. Thus, Mr John Taylor shows the inches, cubits, and axes to answer precisely, although he took as his standard a totally different diameter of the earth from Professor Smyth. The diameter of the earth at 30° of latitude—the geographical position of the Great Pyramid—is some seventeen miles, or more exactly 17·652 miles, or above 1,100,000 inches, longer than at the poles. But Mr Taylor fixed upon this diameter of the earth at latitude 30° —and not, like Professor Smyth, upon its polar diameter—as the standard for the metrological linear measures of the Great Pyramid; and yet, though the standard was so different, he found, like Mr Smyth, 500 millions of inches also in his axis, and 20 millions of cubits also.* The resulting figures appear to fit equally as well for the one as for the other. Perhaps they answer best on Mr Taylor's scheme. For Mr Taylor maintained that the diameter of the earth before the Flood, at this selected point of 30°, was less by nearly 27 miles from what it was subsequently to the Flood,† and is now;—a point by which he accounts for otherwise unaccountable circumstances in the metrological doctrines which have been attempted to be connected with the Great Pyramid. For while Mr Taylor believes the Sacred Cubit to be 24·88, or possibly 24·90 British inches, he holds the new Pyramidal cubit to be 25 inches in full; and the Sacred and Pyramidal cubits to be different, therefore, from each other, though both inspired. In explanation of this startling difference in two measures supposed to be equally of sacred‡ origin, Mr Taylor observes—"The smaller (24·88) is the Sacred Cubit which measured the diameter of the Earth *before* the Flood; the one by which Noah measured the Ark, as tradition says; and the one in accordance with which all the interior works of the Great Pyramid were constructed.* The larger (25) is the Sacred Cubit of the *present* Earth, according to the standard of the Great Pyramid when it was completed."

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* "The diameter of the earth, according to the measures taken at the Pyramids, is 41,666,667 English feet, or 500,000,000 inches." (See "The Great Pyramid," p. 75.) "Dividing this number by 20 millions, we obtain the measure of 25 (English) inches for the Sacred Cubit" (p. 67).

† "When" (says Mr Taylor, p. 91) "the *new* Earth was measured in Egypt after the Deluge, it was found that it exceeded the diameter of the *old* Earth by the difference between 497,664,000 inches and 500,000,000 inches; that is, by 2,336,000 inches, equal to 26·868 miles."

‡ *Alleged Sacred Character of the Scottish Yard or Ell Measure.*—Professor Smyth tries to show (iii. 597), that if Britain stands too low in his metrological testing of the European kingdoms and races, its "low entry is due to accepting the yard for the country's popular measure of length." But long ago the "divine" origin of the Scottish ell—as in recent times the divine origin of the so-called pyramidal cubit and inch—was pleaded rather strenuously. For when, in the 13th century, Edward I. of England laid before Pope Boniface his reasons for attaching the kingdom of Scotland to the Crown of England, he maintained, among other arguments, the justice and legality of this appropriation on the ground that his predecessor King Athelstane, after subduing a rebellion in Scotland under the auspices of St John of Beverley,

Surely such marked diversities and contradictions, and such strange hypothetical adjustments and re-adjustments of the data and calculations, entirely upset the groundless and extraordinary theory of the base of the pyramid being a standard of linear measurement; or a segment of any particular axis of the earth; or a standard for emitting a system of new inches and new cubits;—seeing, on the one hand, more particularly, that the basis line of the pyramid is still itself an unknown and undetermined linear quantity, as is also the polar axis of the earth of which it is declared and averred to be an ascertained, determined, and measured segment.

M. Pancton, in 1780, wrote a work in which he laid down the base side of the pyramid as 8754 inches; maintained, like Mr Taylor and Mr Smyth, that this length was a standard of linear measures; found it to be the measure of a portion of a degree of the meridian, such degree being itself the 360th part of a circle;—and apparently the calculations and figures answered as well as

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prayed that through the intervention of that saint, it "might be granted to him to receive a visible and tangible token by which all future ages might be assured that the Scots were rightfully subject to the King of England. His prayer was granted in this way: Standing in front of one of the rocks at Dunbar, he made a cut at it with his sword, and left a score which proved to be the *precise* length of an ell, and was adopted as the regulation test of that measure of length." This legend of the "miraculously created ell-wand standard" was afterwards duly attested by a weekly service in the Church of St John of Beverley. (See Burton's "History of Scotland," ii. 319.) In the official account of the miracle, as cited by Rymer, it is declared that during its performance the rock cut like butter or soft mud under the stroke of Athelstane's sword. "Extrahens gladium de vaginâ percussit in cilicem, quæ adeo penetrabilis, Dei virtute agente, fuit gladio, quasi eâdem horâ lapis butirum esset, vel mollis glarea; . . . et usque ad presentem diem, evidens signum patet, quod Scoti, ab Anglis devicti ac subjugata; monumento tali evidenter cunctis adeuntibus demonstrante." (Foedera, tom. i. pars ii. 771.)

*Eslewhere (p. 45) Mr Taylor corroborates Sir Isaac Newton's opinion that the *working* cubit by which the Pyramid was built was the cubit of Memphis.

when the measurement was declared to be 9142 inches, and the line, not a segment of an arc of the circumference of the earth, but a segment of the polar axis of the earth; for De l'Isle lauds Pancton's meridian degree theory as one of the wondrous efforts of human genius, or (to use his own words) "as one of the chief works of the human mind!" Yet the errors into which Pancton was led in miscalculating the base line of the Pyramid as 8754 inches, and the other ways he was misled, are enough, suggests Professor Smyth, "to make poor Pancton turn in his grave."

SIGNIFICANCE OF CYPHERS AND FIVES.

M. Pancton, Mr Taylor, and those who have adopted and followed their pyramid metrological ideas, seem to imagine that if, by multiplying one of their measures or objects, they can run the calculation out into a long tail of terminal 0's, then something very exact and marvellous is proved. "When" (upholds Mr Taylor), "we find in so complicated a series of figures as that which the measures of the Great Pyramid and of the Earth require for their expression, *round numbers* present themselves, or such as leave no remainder, we may be sure we have arrived at *primitive* measures." But many small objects, when thus multiplied sufficiently, give equally startling strings of 0's. Thus, if the polar axis of the earth be held as 500,000,000 inches, and Sir Isaac Newton's "Sacred Cubit" be held as 24·82 inches—then the long diameter of the brim of the lecturer's hat, measuring 12·4 inches, is 1-20,000,000th of the earth's polar axis; a page of the print of the Society's Transactions is 1-60,000,000th of the same; a print page of Professor Smyth's book, 6·2 inches in length, is 1-80,000,000th of this "great standard;" &c., &c.

Professor Smyth seems further to think that the figure or number "five" plays also a most important symbolical and inner part in the configuration, structure, and enumeration of the Great Pyramid. "The pyramid" (says he) "embodies in a variety of ways the importance of five." It is itself "five-angled, and with its plane a five-sided solid, in which everything went by fives, or numbers of fives and powers of five." "With five, then, as a number, times of five, and powers of five, the Great Pyramid contains a mighty system of consistently subdividing large quantities to suit human happiness." To express this, Mr Smyth suggests the new noun "fiveness." But it applies to many other matters as strongly, or more strongly, than to the Great Pyramid. For instance, the range of rooms belonging to the Royal Society is "five" in number; the hall in which it meets has five windows; the roof of that hall is divided into five transverse ornamental sections; and each of these five transverse sections is subdivided into five longitudinal ones; the books at each end of the hall are arranged in ten rows and six sections—making sixty, a multiple of five; the official chairs in the hall are ten in number, or twice five; the number of benches on one side for ordinary fellows is generally five; the office-bearers of the Society are twenty-five in number, or five times five; and so on. These arrangements were doubtless, in the first instance, made by the Royal Society without any special relation to "fiveness," or the "symbolisation" of five; and there is not the slightest ground for any belief that the apparent "fiveness" of anything in the Great Pyramid had a different origin.

Minuteness of Modern Practical Standards or Gauges.

In all these "standards" of capacity and length alleged to exist about the Great Pyramid, not only are the theoretical and actual sizes of the supposed "standards" made to vary in different books—which it is impossible for an actual "standard" to do,—but the evidences adduced in proof of the conformity of old or modern measures with them is notoriously defective in complete aptness and accuracy. Measures, to be true counterparts, must, in mathematics, be not simply "near" or "very near," which is all that is generally and vaguely claimed for the supposed pyramidal proofs; but they must be entirely and *exactly* alike, which the pyramidal proofs fail altogether in being. Mathematical measurements of lines, sizes, angles, &c., imply exactitude and not mere approximation; and without that exactitude they are not mathematical, and —far more—are they not "superhuman" and "inspired."

Besides, it must not be forgotten that our real *practical* standard measures are infinitely more refined and many thousand-fold more delicate than any indefinite and equivocal measures alleged to be found in the pyramid by even those who are most enthusiastic in the pyramidal metrological theory. At the London Exhibition in

1851, that distinguished mechanician, Mr Whitworth, of Manchester, was the first to show the possibility of ascertaining by the sense of touch alone the one-millionth of an inch in a properly-adjusted standard of linear measure; and in his great establishment at Manchester they work and construct machinery and tools of all kinds with differences in linear measurements amounting to one ten-thousandth of an inch. The standards of the English inch, &c., made by him for the Government—and now used by all the engine and tool makers, &c., of the United Kingdom—lead to the construction of machinery, &c., to such minute divisions; and the adoption of these standards has already effected enormous saving to the country by bringing all measured metal machinery, instruments, and tools, wherever constructed and wherever afterwards applied and used, to the same identical series of mathematical and precise gauges.

The Sabbath, &c., Typified in the Pyramid.

The communication next discussed some others amongst the many and diversified matters which Professor Smyth fancifully averred to be typified and symbolised in the Great Pyramid.

One, for example, of the chambers in the Great Pyramid—the so-called Queen's Chamber—has a roof composed of two large blocks of stone leaning against each other, making a kind of slanting or double roof. This double roof, and the four walls of the chamber count six, and typify, according to Professor Smyth, the six days of the week, whilst the floor counts, as it were, a seventh side to the room, "nobler and more glorious than the rest," and typifying something, he conceives, of a "nobler and more glorious order"—namely, the Sabbath. It is surely difficult to fancy anything more strange than this strange idea. In forming this theory liberties are also confessedly taken with the floor in order to make it duly larger than the other six sides of the room, and to do so he theoretically lifts up the floor till it is placed higher than the very entrance to the chamber; for originally the floor and sides are otherwise too nearly alike in size to make a symbolic *seven*-sided room with one of the sides proportionally and properly larger than the other six sides. Yet Professor Smyth holds that, in the above typical way, he has "shown," or indeed "proved entirely," that the Sabbath had been heard of before Moses, and that thus he finds unexpected and confirmatory light of a fact which, he avers, is of "extraordinary importance, and possesses a ramifying influence through many departments of religious life and progress."

He believes, also, that the corner-stone—so frequently alluded to by the Psalmist and the Apostles as a symbol of the Messiah—is the head or corner-stone of the Great Pyramid, which, though long ago removed, may yet possibly, he thinks, be discovered in the Cave of Machpelah; though how, why, or wherefore it should have found its way to that distant and special locality is not in any way solved or suggested.

Pyramid alleged to be a Superhuman, and more or less an Inspired Metrological Erection.

Professor Smyth holds the Great Pyramid to be in its emblems and intentions and work "superhuman;" as "not altogether of human origination; and in that case whereto" (he asks) "should we look for any human assistance to men but from Divine inspiration?" "Its metrology is," he conceives, "directed by a higher Power" than man; its erection "directed by the *fiat* of Infinite Wisdom;" and the whole "built under the direction of chosen men divinely inspired from on high for this purpose."

If of this Divine origin, the work should be absolutely perfect; but, as owned by Professor Smyth, the structure is not entirely correct in its orientation, in its squareness, &c., &c.—all of them matters proving that it is human, and not superhuman. It was, Professor Smyth further alleges, intended to convey standards of measures to all times down to, and perhaps beyond, these latter days, "to herald in some of those accompaniments of the promised millennial peace and goodwill to all men." Hence, if thus miraculous in its foreseen uses, it ought to have remained relatively perfect till now. But "what feature of the pyramid is there" (asks Professor Smyth) "which renders at once in its measurements in the present day its ancient proportions? None." If the pyramid were a miracle of this kind, then the Arabian Caliph El Mamoon so far upset the supposititious miracle a thousand years ago—(of course he could not have done so provided the miracle had been truly Divine)—when he broke into the King's Chamber and unveiled its contents; inasmuch as the builders, according to Professor Smyth, intended to conceal its secrets for the benefit of these latter times, and for this purpose had left a mathematical sign of a cross joint on the floor of the gallery, by which some man in the distant future, visiting the interior, should detect the entrance to the chambers; and which secret sign Professor Smyth himself was, as he believes, the first to discover two years ago. The secret, however, if any such was placed there, for the detection of the entrance to the interior chambers, has been thus discovered some 1000 years too late for the evolution of the alleged miraculous arrangement. In relation to the Great Pyramid, as to other things, we may be sure that God does not teach by the medium of miracle anything that the unaided intellect of man can find out; and we must beware of wrongously and disparagingly attributing to Divine inspiration and aid, things that are imperfect and human.

The communication concluded by a long series of remarks, in which it was pointed out that at the time at which the Great Pyramid was built, probably about 4000 years ago, mining, architecture, astronomy, &c., were so advanced in various parts of the East as to present no obstacle in the way of the erection of such magnificent mausoleums as the colossal Great Pyramid and its other congener pyramids undoubtedly are.

2. Note on the Occasional Occurrence of the Musculus Rectus Thoracis in Man. By Professor Turner.

During the last winter session, I communicated to the Society* a paper on the "Musculus Sternalis," in which I argued that the longitudinally arranged muscle, occasionally found superficial to the sternal fibres of origin of the pectoralis major, was not, as anatomists have usually described it, homologous with the anterior or thoracic fibres of the mammalian rectus, but belonged to another group of muscles.

Since that time I have met with two subjects in the dissecting-room, in each of which a longitudinal muscle occurred, lying in contact with the outer surface of the anterior extremities of the upper true ribs, and beneath the fibres of the pectoral muscle, which is, from the position, direction, and connection of its fibres, I believe to be regarded as homologous with the thoracic end of the mammalian rectus. Both subjects were males. In one a longitudinal ribbon-shaped muscle arose by a thin expanded tendon, from the upper border of the fifth left rib, immediately internal to the attachment of the serratus magnus. The innermost part of its tendon was continuous with the membrane covering the internal intercostal muscle, and was attached to the rib at the junction of its osseous and cartilaginous portions. From the anterior surface of the fourth rib, close to the origin of the serratus magnus, a second and smaller origin proceeded. The muscle ascended superficial to the osseous parts of the third and second ribs, as high as the first rib, into which it was inserted immediately external to the tendon of attachment of the subclavius muscle. The muscle was 6 inches long and $\frac{5}{8}$ths of an inch broad at its widest part. The breadth of the fifth rib alone separated it from the upper attachment of the rectus abdominis. A corresponding muscle existed on the right side.

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* Proceedings, 21st January 1867, and Journal of Anatomy and Physiology, May 1867.

In the other subject, a longitudinal muscle—on the right side only—arose beneath the pectoralis minor from the upper border of the fourth rib two inches to the outer side of the junction of its bone and cartilage. It ascended superficial to the third and second ribs, to be inserted into the bony part of the first rib, $\frac{3}{4}$ths of an inch external to the attachment of the subclavius.

In the cat, the otter, the beaver, the porcupine, and various other Mammalia, the rectus muscle extends as high as the first rib, into which it is inserted, and in this respect the arrangement may be compared with that of the occasional human muscles just described.

In these animals, however, the thoracic and abdominal parts of the rectus are directly continuous with each other, whilst in the human subject a break, corresponding in the first specimen to the fifth rib, and in the second, to the fifth rib and fourth intercostal space, occurred; but this break may be regarded as comparable to one of those transverse tendinous intersections, invariably found in the abdominal part of the human rectus, and which exist also in the recti of the greater number of the Mammalia. These specimens seem to present as complete a representation of the mammalian arrangement, though in a somewhat different form, as is afforded by the case recorded by Kaau Boerhaave, to which I alluded in my former paper, and they support the opinion I ventured to throw out in that communication, that the supracostal muscle described by Mr John Wood (*Proc. Roy. Soc. London*, June 15, 1865), is homologous with the pectoral end of the mammalian rectus.

During the past year, Mr Wood has described another specimen of the supra-costal muscle (*Proc. Roy. Soc. London*, May 23, 1867) which passed from the third rib upwards to the first rib and cervical fascia; Dr Roberts (*Liverpool Medical and Surgical Reports*, October 1867), one which extended from the fourth to the first rib; and Bochdalek, Jun. (Virchow's *Archiv.* 18th November 1867), under the name of M. supra-costalis anterior, has described and figured one, which also passed from the fourth to the first rib; but by none of these anatomists is any reference made to the probability of its being homologous with the pectoral end of the mammalian rectus.

The specimens which I have now recorded apparently correspond to the supra-costal muscles of these writers; and the arrangement more especially of the first, in which the muscle extended as low as the fifth rib, and was separated from the rectus only by the breadth of that rib, justifies me, I think, in the conclusion I have come to respecting the homology of the muscle.